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Modified Takabayasi String in Bulk Viscous Bianchi Type-III Cosmology: Dynamics in the Presence of Variable Λ

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Abstract:

In the current work, the solutions for Bianchi type III cosmological models in the presence of bulk viscous fluid are derived using a modified Takabayasi string of the form$\rho = (1 + \omega)\lambda + \Lambda$. In order to achieve a realistic model, we make the assumption that there is a relationship between the metric potential, denoted as X_2, and another variable, denoted as X_3, raised to the power of n. We examined the significance of the Λ word in the cosmic evolution. In this study, we examined two distinct scenarios for the cosmological model, namely case i) where ω tends towards infinity, and case ii) where ω tends towards zero. In both situations, we have determined the physical meaning of the cosmological term Λ in a quadratic form, specifically $\Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2$. The physical and geometrical components of the model are thoroughly examined for both scenarios, and their associated solutions are discussed. It has been noted that the resulting models align with current findings and literature that support the idea of the universe's acceleration. In addition, we have generated diagrams to facilitate a comprehensive analysis of each individual scenario.

Keywords: Modified Takabayasi String, Bianchi Type III Spacetime, Bulk Viscosity, cosmological Constant Λ.

1. Introduction

During the birth of the universe, string theory emerged. According to this theory, cosmic strings are formed after the big bang explosion during phase transitions, when the temperature drops below a critical point. This concept has been discussed by Zel'dovich, Kibble, and Vilenkin [46, 19, 44]. Stachel (reference: [38]) has examined the properties of strings that have no mass. Letelier [21] investigated the properties of enormous strings composed of a geometric thread with particles attached along its length. These strings, known as cloud strings, are a generalization of Takabayasi's realistic model of strings [40], which are referred to as p strings [20]. Letelier [21] provided the following examples of equations of state for strings:

The geometric string is defined by the equation $\rho = \lambda$, where ρ_p represents the rest energy density of the cloud. The equation for Takabayasi strings is given by $\rho = (1 + \omega)\lambda$. The equation is defined as $\rho = \rho(\lambda)$, where $\rho_p = \rho - \lambda$. Here, ω is a positive constant. The modified equation of state (EoS) for a cloud of string is given by the expression $\rho = (1 + \omega)\lambda + \Lambda(t)$.

The symbol $\Lambda(t)$ represents the cosmological constant. The physical and geometrical components of the model are thoroughly examined for both scenarios, and their associated solutions are discussed. It has been noted that the resulting models align with current findings and literature that support the idea of the universe's accelerating model. In addition, we have generated diagrams to facilitate a comprehensive analysis of each individual situation.

Banerjee et al. [2] have studied some cosmological solutions of massive strings for Bianchi type I space-time while Rahman et al. [32] have investigated the case of geometric string, Takabayasi string and barotropic string in their research work. Bianchi Type-III cosmological models with gravitational constant G and the cosmological constant Λ have been studied by [36]. Yadav et al. [45] have discussed some Bianchi Type III string cosmological models with bulk Viscosity and while Bali and Pradhan [1] have studied Bianchi Type III string cosmological models with time dependent bulk viscosity.

Tiwari and Sharma [42] have discussed non existence of shear in Bianchi Type-III string cosmological models with bulk viscosity and time dependent Λ term and Singh [37] investigated anisotropic Bianchi V universe with magnetic field and bulk viscous fluid by taking different string models like geometric (Nambu string), Takabayasi string (p-string) and Reddy string and discussed the physical and geometrical aspects of each string model. Inhomogeneous Bianchi type-VIo String Cosmological Model for Stiff Perfect Fluid Distribution in General Relativity has been discussed by [35].

Nojiri and Odinstov [27] investigated inhomogeneous equation of state of the universe: Phantom era, future singularity and crossing the phantom barrier. Cosmological

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scenarios with a time-varying Λ were proposed by several researchers. A number of models with different decay laws for the variation of cosmological term were investigated during last two decades [8, 30,7,22,23].

The present universe is subject to an acceleration, which can be explained in terms of an ideal fluid (dark energy) weakly interacting with usual matter and which has an uncommon EOS. The origin of this dark energy is really dark: the proposed explanations vary from the modifications of gravity to the introduction of new fields with really strange properties. In the study of celebrated dark energy problem (for a review, see [9]) the ideal fluid with specific (sometimes, strange) EOS remains to be the simplest possibility for the description of the current cosmic acceleration. Various examples of ideal fluid EOS may be considered for this purpose: constant EOS with negative pressure, imperfect EOS [6], General EOS [26, 39], inhomogeneous EOS [27, 28, 5, 3, 33] and so on for extensive review, see [9]. Moreover, general relativity with ideal fluid of any type may be rewritten in the equivalent form [5] as modified gravity [24]. Of course, the introduction of ideal fluid with complicated EOS is somehow phenomenological approach because no explanation of the origin of such dark fluid follows.

Brevik et al. [4] studied a model where there is an ideal fluid with an inhomogeneous equation of state of the form $p = \omega(t)\rho + \Lambda(t)$, in which the parameters $\omega(t)$ and $\Lambda(t)$ depend linearly on time. One of these proposals deals with dark fluids, which possess an inhomogeneous EOS that may depend on time [27,29,5]. Gadbail et al. [10] have studied Interaction of divergence-free deceleration parameter in Weyl-type $f(Q,T)$ gravity while Kumar et al. [16] have investigated Two Fluids Cosmological Model in $(2+1)$-Dimensional Saez-Ballester Scalar-Tensor Theory of Gravitation. Talole et al. [41] have discussed Viscous Modified Ghost Scalar Field Dark Energy Models with Varying G and Kumbhare et al. [15] have studeied Strange Quark Matter attached to String Cloud in General Relativity under 5D space-time.

Islam et al. [12] investigated Gravitational Model of Compact Spherical Reissner-Nordström-Type Star Under $f(R,T)$ Gravity and Georgiev et al. [11] have brief about the Two Dimensional Integral Inequalities on Time Scales. Ramtekkar et al. [31] have discussed FRW Viscous Modified Cosmic Chaplygin Gas Cosmology with Variable Cosmological Constant Λ, Khadekar et al. [17] Modified Chaplygin gas with bulk viscous cosmology in FRW (2+ 1)-dimensional spacetime and Islam et al. [13] (2+ 1) dimensional cosmological models in $f(R,T)$ gravity with $\Lambda(R,T)$.

In 2008, Jamila and Rashid [14] have investigated the model of dark energy interacting with dark matter by choosing inhomogeneous equation of state for dark energy and a nonlinear interaction term for the underlying interaction. The equations of state have dependencies either on the energy densities, the redshift, the Hubble parameter or the bulk viscosity.

By considering these possibilities the authors have derived the effective equations of state for the dark energy and studied each case in detail. Recently, Tripathi [43] have studied Inhomogeneous Bianchi Type I Cosmological Model for Stiff Perfect Fluid Distribution. Also, FRW viscous fluid cosmological model with time-dependent inhomogeneous equation of state have been studied by [18].

In this study, we have examined the modified Takabayasi string, represented by the equation $\rho = (1+\omega)\lambda + \Lambda$, in the context of Bianchi III spacetime. Specifically, we have focused on the case where the parameter ω approaches its limiting value in the presence of a bulk viscous fluid denoted by ζ. The structure of the work is as follows: The concise overview of the introduction is offered in section 1. Section 2 provides a description of the metric and field equations.

Section 3 provides a summary of the paper's findings, specifically Case i) when ω approaches infinity, and Case ii) when ω approaches zero. The results of our research are detailed in section (4). The study concludes with final thoughts in the last section.

2. The Metric and it's Field Equations

Let us consider the Bianchi type-III space-time of the metric as

$$ds^2 = -dt^2 + X_1^2 dx^2 + X_2^2 e^{2\alpha x} dy^2 + X_3^2 dz^2 \dots\dots\dots\dots\dots\dots\dots (1)$$

Here, we assume the energy momentum tensor with bulk stress in the given form as,

$$T_{\mu\nu} = \rho u_\mu u_\nu - \lambda x_\mu x_\nu - \zeta\Theta\left(u_\mu u_\nu + g_{\mu\nu}\right) \dots\dots\dots\dots\dots\dots\dots\dots. (2)$$

Where, ρ, ζ and λ are the energy density, coefficient of bulk viscosity and the string tension respectively. u_μ is the four velocity vector of the fluid satisfying the relation $u_\mu u^\nu = -1$ and the string is assumed in the Z-direction.

We consider modified EoS for Takabayasi string of the following form as given,

$$\rho = (1+\omega)\lambda + \Lambda(t) \dots\dots\dots\dots\dots\dots\dots. (3)$$

Here, we take the cosmological term Λ in quadratic form as

$$\Lambda = \Lambda_0 + \Lambda_1 H + \Lambda_2 H^2 \dots\dots\dots\dots\dots\dots\dots\dots (4)$$

The Einsteins field equations are given as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -T_{\mu\nu} \ldots\ldots\ldots\ldots\ldots (5)$$

The field equation (5) provides the solution for the metric Eq. (1) and energy momentum tensor (2) in the co-moving system of coordinates are

$$\frac{\dot{X}_1\dot{X}_2}{X_1X_2} + \frac{\dot{X}_2\dot{X}_3}{X_2X_3} + \frac{\dot{X}_1\dot{X}_3}{X_1X_3} - \frac{\alpha^2}{X_1^2} = \rho \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$\frac{\ddot{X}_2}{X_2} + \frac{\ddot{X}_3}{X_3} + \frac{\dot{X}_2\dot{X}_3}{X_2X_3} = \zeta\Theta \ldots\ldots\ldots\ldots\ldots\ldots. (7)$$

$$\frac{\ddot{X}_1}{X_1} + \frac{\ddot{X}_3}{X_3} + \frac{\dot{X}_1\dot{X}_3}{X_1X_3} = \zeta\Theta \ldots\ldots\ldots\ldots..\ldots. (8)$$

$$\frac{\ddot{X}_1}{X_1} + \frac{\ddot{X}_2}{X_2} + \frac{\dot{X}_1\dot{X}_2}{X_1X_2} - \frac{\alpha^2}{X_1^2} = \lambda + \zeta\Theta \ldots\ldots\ldots..\ldots\ldots\ldots (9)$$

$$\frac{\dot{X}_1}{X_1} - \frac{\dot{X}_2}{X_2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots.. (10)$$

After integrating Eq. (10), we have

$$X_1 = nX_2 \ldots\ldots\ldots\ldots.. (11)$$

where n is the constant of integration.

It is postulated that the expansion Θ in the model is directly proportional to the shear scalar σ, resulting in the following condition as

$$X_2 = X_3^m \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The Einstein field equations from (6) to (9) can be written by,

$$2m\frac{\ddot{X}_3}{X_3} + m(3m-2)\frac{\dot{X}_3{}^2}{X_3^2} - \frac{\alpha^2}{n^2X_3^{2m}} = \lambda + \zeta\Theta \ldots\ldots\ldots\ldots\ldots (13)$$

$$m(m+2)\frac{\dot{X}_3^2}{X_3^2} - \frac{\alpha^2}{n^2X_3^{2m}} = \rho \ldots\ldots\ldots\ldots.\ldots.. (14)$$

$$(m+1)\frac{\ddot{X}_3}{X_3} + m^2\frac{\dot{X}_3^2}{X_3^2} = \zeta\Theta \ldots\ldots\ldots\ldots\ldots.. (15)$$

The metric (1) introduces several defined quantities: the spatial volume denoted as V, the generalized Hubble parameter represented as H, the expansion factor indicated by Θ, the shear scalar denoted as σ^2, the deceleration parameter represented by q, and the anisotropy parameter denoted as A_h,

$$V = R^3 = X_3^{2m+1} \ldots\ldots\ldots\ldots\ldots (16)$$

$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{(2m+1)}{3}\frac{\dot{X}_3}{X_3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

$$\Theta = u^{\mu}_{;\mu} = (2m+1)\frac{\dot{X}_3}{X_3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - \frac{\Theta^2}{3}\right) = \frac{(m-1)^2}{3}\frac{\dot{X}_3^2}{X_3^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (19)$$

$$q = -\frac{R\ddot{R}}{\dot{R}^2} = -\frac{3}{(2m+1)}\frac{\frac{\ddot{X}_3}{X_3}}{\frac{\dot{X}_3^2}{X_3^2}} + \frac{3}{(2m+1)} - 1, \ldots\ldots\ldots\ldots\ldots (20)$$

$$A_h^2 = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 = 2\frac{(m-1)^2}{(2m+1)^2} \ldots\ldots\ldots\ldots\ldots\ldots. (21)$$

3. Solution of the field equations

From the Eq. (13) and Eq. (15), we get the following equation are

$$\lambda = (m-1)\frac{\ddot{X}_3}{X_3} + 2m(m-1)\frac{\dot{X}_3^2}{X_3^2} - \frac{\alpha^2}{n^2 X_3^{2m}} \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

After using by modified equation of state from Eq. (3) and Eq. (14) becomes,

$$\lambda = \frac{1}{(1+\omega)}\left[m(m+2)\frac{\dot{X}_3^2}{X_3^2} - \frac{\alpha^2}{n^2 X_3^{2m}} - \Lambda\right] \ldots\ldots\ldots\ldots.. (23)$$

By equating Eqns. (22) and (23), we have

$$\frac{\ddot{X}_3}{X_3} + \left[2m - \frac{m(m+2)}{(m-1)(1+\omega)}\right]\frac{\dot{X}_3^2}{X_3^2}$$

$$-\frac{\propto^2 \omega}{n^2(m-1)(1+\omega)}\frac{1}{X_3^{2m}} + \frac{\wedge}{(m-1)(1+\omega)} = 0 \ldots\ldots\ldots\ldots..(24)$$

After using the help of Eq. (4), this equation can be reduces to

$$\frac{\ddot{X}_3}{X_3} + \left[2m - \frac{m(m+2)}{(m-1)(\omega+1)} + \frac{\Lambda_2(2m+1)^2}{9(m-1)(\omega+1)}\right]\frac{\dot{X}_3^2}{X_3^2}$$

$$+\frac{\wedge_1(2m+1)}{3(m-1)(\omega+1)}\frac{\dot{X}_3}{X_3} - \frac{\propto^2\omega}{n^2(m-1)(1+\omega)}\frac{1}{X_3^{2m}} + \frac{\wedge_0}{(m-1)(1+\omega)} = 0 \ldots\ldots\ldots\ldots..(25)$$

It is not possible to obtain the exact solution of the above differential equation. Hence, here we discuss the following two limiting cases depending upon ω as follows: Case (i): $\omega \to \infty$ and Case (ii): $\omega \to 0$.

3.1.Case (i): $\omega \to \infty$

When $\omega \to \infty$, the differential equation (25) can be obtained as,

$$\frac{\ddot{X}_3}{X_3} + 2m\frac{\dot{X}_3{}^2}{X_3^2} - \frac{\mathcal{A}_0}{X_3^{2m}} = 0 \ldots\ldots\ldots\ldots (26)$$

where

$$\mathcal{A}_0 = \frac{\alpha^2}{n^2(m-1)}$$

By using the substitution $X_3 = e^{\mathcal{B}}$ in the differential equation Eq. (26),

we get

$$\ddot{\mathcal{B}} + (2m+1)\dot{\mathcal{B}}^2 = \mathcal{A}_0 e^{-2m\mathcal{B}} \ldots\ldots\ldots\ldots..(27)$$

Again, we put $\dot{\mathcal{B}}^2 = p$ in Eq. (27), reduces to,

$$\frac{dp}{d\mathcal{B}} + 2(2m+1)p = 2\mathcal{A}_0 e^{-2m\mathcal{B}} \ldots\ldots\ldots\ldots.(28)$$

After integrating on both side, we have

$$\int \frac{d\mathcal{B}}{\sqrt{\frac{\mathcal{A}_0 e^{-2m\mathcal{B}}}{(m+1)} + c_0 e^{-2(2m+1)\mathcal{B}}}} = (t - t_0) \ldots\ldots\ldots\ldots..(29)$$

where c_0 and t_0 are the constant of integrating.

We are unable to determine the solution to the given differential equation.

Thus, we consider the specific case where $c_0 = 0$. Therefore, the differential equation (29) can be rewritten as

$$\int e^{m\mathcal{B}} d\mathcal{B} = \sqrt{\frac{\mathcal{A}_0}{(m+1)}}(t - t_0) \ldots\ldots\ldots.\ldots\ldots (30)$$

After integrating the Eq. (30) on both side, we have

$$\mathcal{B} = \log X_3$$

$$= \frac{1}{m}\log\left[m\sqrt{\frac{\mathcal{A}_0}{(m+1)}}(t - t_0)\right] \ldots\ldots\ldots\ldots\ldots..(31)$$

After substituting the value of $\mathcal{A}_0$ from Eq. (27) in Eq. (31), we get

$$X_3 = \left[\frac{\alpha m}{n\sqrt{(m^2-1)}}(t - t_0)\right]^{\frac{1}{m}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (32)$$

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3.2. Some physical and geometrical aspects

The expression for energy density ρ, string tension λ, cosmological term Λ and bulk viscosity coefficient ζ are given as

$$\rho = \frac{(2m+1)}{m^2}\frac{1}{(t-t_0)^2} \dots\dots\dots\dots\dots (33)$$

$$\lambda = 0 \dots\dots\dots\dots\dots (34)$$

$$\Lambda = \Lambda_0 + \frac{\Lambda_1(2m+1)}{3m}\frac{1}{(t-t_0)} + \frac{\Lambda_2(2m+1)^2}{9m^2}\frac{1}{(t-t_0)^2} \dots\dots\dots\dots\dots (35)$$

$$\zeta = \frac{1}{m(2m+1)}\frac{1}{(t-t_0)} \dots\dots\dots\dots\dots\dots (36)$$

Similarly, the expression for spatial volume V, Hubble parameter H, expansion scalar Θ, shear scalar σ^2 are given follows

$$V = \left[\frac{\alpha m(t-t_0)}{n\sqrt{(m^2-1)}}\right]^{\frac{2m+1}{m}} \dots\dots\dots\dots (37)$$

$$H = \frac{(2m+1)}{3m}\frac{1}{(t-t_0)} \dots\dots\dots\dots (38)$$

$$\Theta = \frac{(2m+1)}{m}\frac{1}{(t-t_0)} \dots\dots\dots (39)$$

$$\sigma^2 = \frac{(m-1)^2}{3m^2}\frac{1}{(t-t_0)^2} \dots\dots\dots (40)$$

$$q = \frac{(m-1)}{(2m+1)} \dots\dots\dots\dots (41)$$

$$A_h = \frac{2(m-1)^2}{(2m+1)^2} \dots\dots\dots\dots (42)$$

3.3. Case (ii): $\omega \to 0$

In this case, Eq. (3) reduces to $\rho = \lambda + \Lambda(t)$ which is a modified geometric string and hence, Eq. (25) reduces to,

$$\frac{\ddot{X}_3}{X_3} - Z_1\frac{\dot{X}_3^2}{X_3^2} + Z_2\frac{\dot{X}_3}{X_3} + Z_3 = 0 \dots\dots\dots (43)$$

where

$$Z_1 = \left[m(4-m) - \frac{\Lambda_2(2m+1)^2}{9}\right]\frac{1}{(m-1)},$$

$$Z_2 = \frac{\Lambda_1(2m+1)}{3(m-1)},$$

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$$Z_3 = \frac{\Lambda_0}{(m-1)}.$$

After solving the Eq. (43), we get

$$X_3 = d_2 \exp\left[\frac{Z_2 t}{2(Z_1-1)}\right]\left[\sec\left(Z_4\left(d_1+\frac{t}{2}\right)\right)\right]^{\frac{1}{Z_1-1}} \dots\dots\dots\dots..(44)$$

where, d_1 and d_2 are the constant of integration and

$$Z_4 = \sqrt{4Z_3 - 4Z_1Z_3 - Z_2^2}.$$

3.3.1. Some physical and geometrical aspects

The expressions for energy density ρ, string tension λ, cosmological term Λ, coefficient of viscosity ζ, the spatial volume V, Hubble parameter H, expansion scalar Θ, shear scalar σ^2, deceleration parameter q and anisotropic parameter A_h as follows,

$$\rho = \frac{m(m+2)}{4(Z_1-1)^2}\left[Z_4 \tan\left[Z_4\left(d_1+\frac{t}{2}\right)\right]+Z_2\right]^2$$

$$-\frac{\alpha^2}{n^2}\frac{1}{\left[d_2 \exp\left[\frac{Z_2 t}{2(Z_1-1)}\right]\left[\sec\left(Z_4\left(d_1+\frac{t}{2}\right)\right)\right]^{\frac{1}{Z_1-1}}\right]^{2m}} \dots\dots(45)$$

$$\lambda = \frac{Z_4^2(m-1)}{4(Z_1-1)}\sec^2\left[Z_4\left(d_1+\frac{t}{2}\right)\right]$$

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$$+\frac{(m-1)(2m+1)}{4(Z_1-1)^2}\left[Z_4\tan\left[Z_4\left(d_1+\frac{t}{2}\right)\right]+Z_2\right]^2$$
$$-\frac{\alpha^2}{n^2}\frac{1}{\left[d_2\exp\left[\frac{Z_2 t}{2(Z_1-1)}\right]\left[\sec\left(Z_4\left(d_1+\frac{t}{2}\right)\right)\right]^{\frac{1}{Z_1-1}}\right]^{2m}} \ldots\ldots.(46)$$

$$\Lambda=\Lambda_0+\frac{\Lambda_1(2m+1)}{6(Z_1-1)}\left[Z_4\tan\left[Z_4\left(d_1+\frac{t}{2}\right)\right]+Z_2\right]$$
$$+\frac{\Lambda_2(2m+1)^2}{36(Z_1-1)^2}\left[Z_4\tan\left[Z_4\left(d_1+\frac{t}{2}\right)\right]+Z_2\right]^2 \ldots\ldots\ldots.(47)$$

$$\zeta=\frac{(m+1)Z_4^2}{2(2m+1)}\frac{\sec^2\left[Z_4\left(d_1+\frac{t}{2}\right)\right]}{\left[Z_4\tan\left[Z_4\left(d_1+\frac{t}{2}\right)\right]+Z_2\right]}$$
$$+\frac{(m^2+m+1)}{2(2m+1)(Z_1-1)}\left[Z_4\tan\left[Z_4\left(d_1+\frac{t}{2}\right)\right]+Z_2\right] \ldots\ldots.(48)$$

$$V=\left[d_2\exp\left[\frac{Z_2 t}{2(Z_1-1)}\right]\left[\sec\left(Z_4\left(d_1+\frac{t}{2}\right)\right)\right]^{\frac{1}{Z_1-1}}\right]^{2m+1} \ldots\ldots.(49)$$

$$H=\frac{(2m+1)}{6(Z_1-1)}\left[Z_4\tan\left[Z_4\left(d_1+\frac{t}{2}\right)\right]+Z_2\right] \ldots\ldots.(50)$$

$$\Theta=\frac{(2m+1)}{2(Z_1-1)}\left[Z_4\tan\left[Z_4\left(d_1+\frac{t}{2}\right)\right]+Z_2\right] \ldots\ldots\ldots(51)$$

$$\sigma^2=\frac{(m-1)^2}{12(Z_1-1)^2}\left[Z_4\tan\left[Z_4\left(d_1+\frac{t}{2}\right)\right]+Z_2\right]^2 \ldots\ldots.(52)$$

$$q=\frac{-3Z_4^2(Z_1-1)}{(2m+1)}\frac{\sec^2\left[Z_4\left(d_1+\frac{t}{2}\right)\right]}{\left[Z_4\tan\left[Z_4\left(d_1+\frac{t}{2}\right)\right]+Z_2\right]^2}-1, \ldots\ldots\ldots.(53)$$

$$A_h=\frac{2(m-1)^2}{(2m+1)^2} \ldots\ldots\ldots\ldots\ldots.(54)$$

4. Results and Discussion

Here, we have derived solutions to the Einstein field equations for two specific conditions:

Case (i): $\omega \to \infty$

- From graph (1), it can be noted that as the variable t grows, the scale factors likewise increase, whereas the energy density ρ, cosmological term Λ, and Hubble parameter H (as shown in figures (3), (5), and (7)) drop with increasing t.
- The criteria for energy density, $\rho > 0$, is fulfilled. From equation (35), it is evident that the value of λ is zero.

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- The cosmological term Λ is positively correlated with time t and exhibits a decreasing trend.

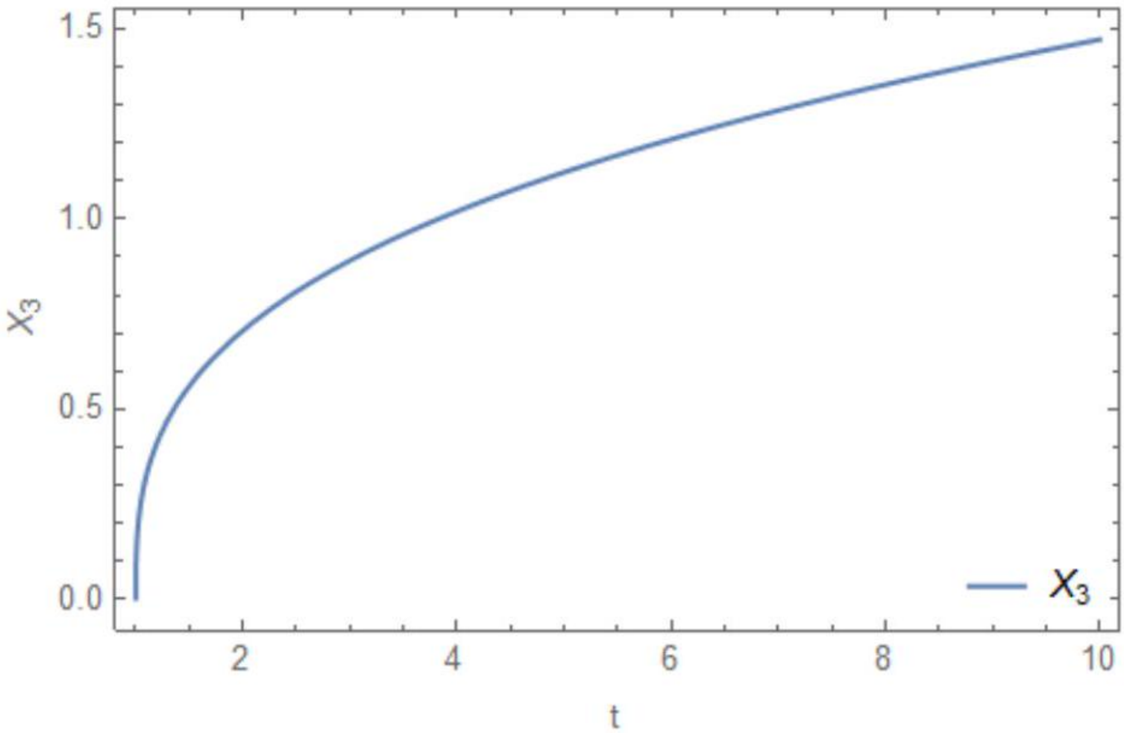

Figure 1: Case (i) $\to \infty$: The relation between X_3 and time t, $m = 3$, $\alpha = 1$, $n = 3$, $\Lambda_0 = 3$, $\Lambda_1 = 1$, $\Lambda_2 = 2$, $t_0 = 1$.

Case (ii): $\omega \to 0$, here we discuss the modified geometric string.

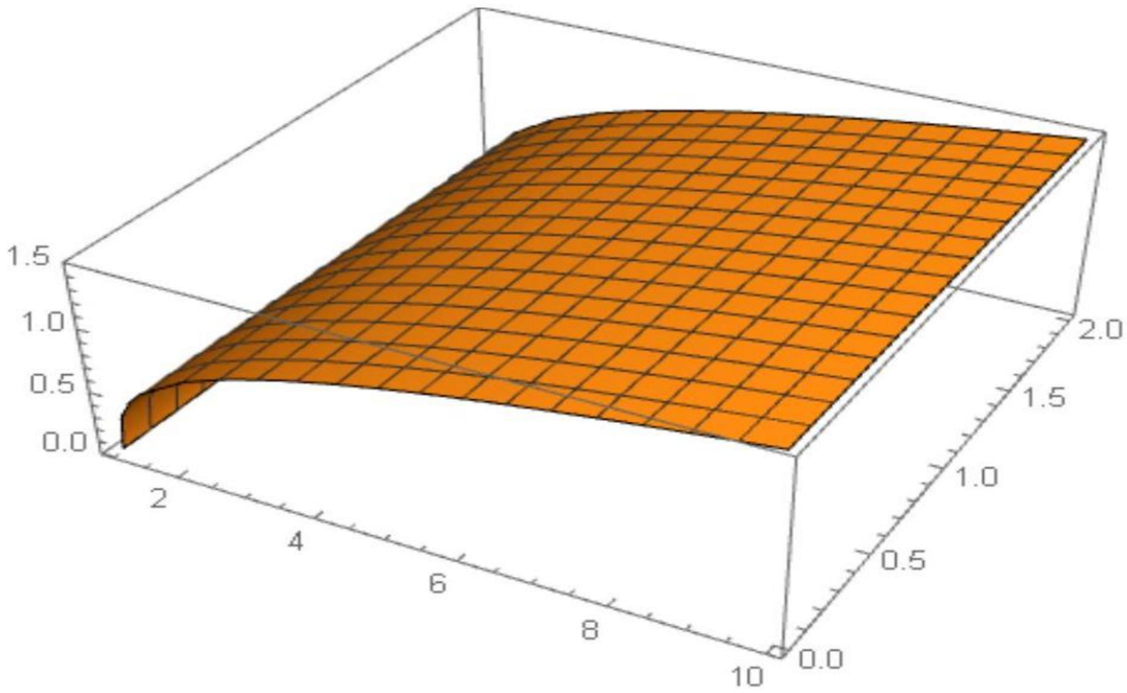

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Figure 2: Case (i) $\rightarrow \infty$: The relation between X_3 and Cosmic time t in 3-Dimensional, $m = 3,\ \alpha = 1,\ n = 3,\ \Lambda_0 = 3,\ \Lambda_1 = 1,\ \Lambda_2 = 2,\ t_0 = 1.$

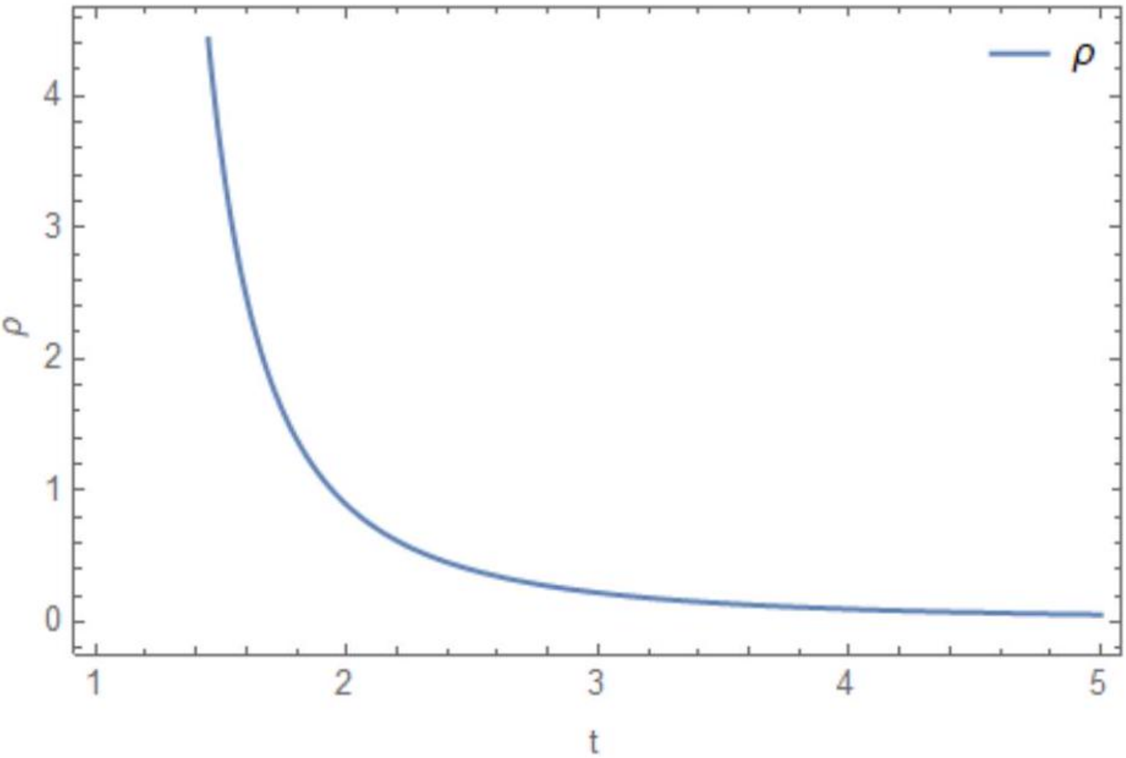

Figure 3: Case (i) $\rightarrow \infty$: The relation between energy density ρ and Cosmic time t, $m = 3,\ \alpha = 1,\ n = 3,\ \Lambda_0 = 3,\ \Lambda_1 = 1,\ \Lambda_2 = 2,\ t_0 = 1.$

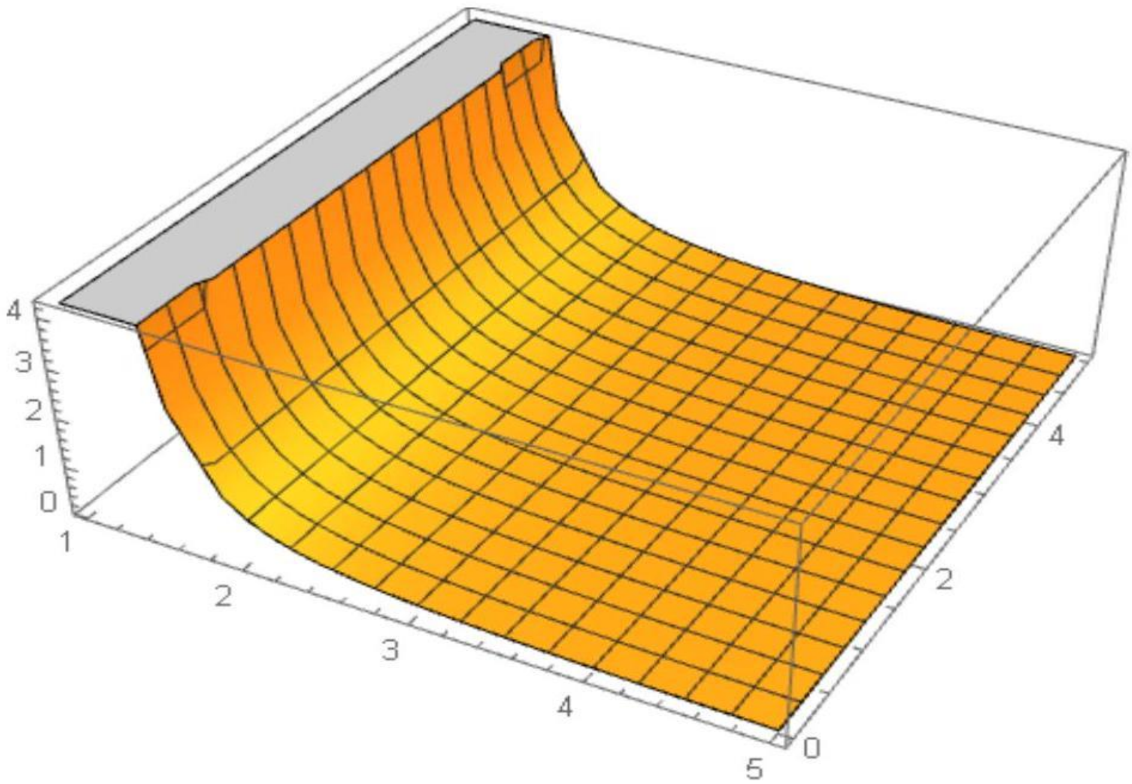

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Figure 4: Case (i) $\rightarrow \infty$: The relation between energy density ρ and Cosmic time t in 3-Dimensional, $m = 3,\ \alpha = 1,\ n = 3,\ \Lambda_0 = 3,\ \Lambda_1 = 1, \Lambda_2 = 2,\ t_0 = 1.$

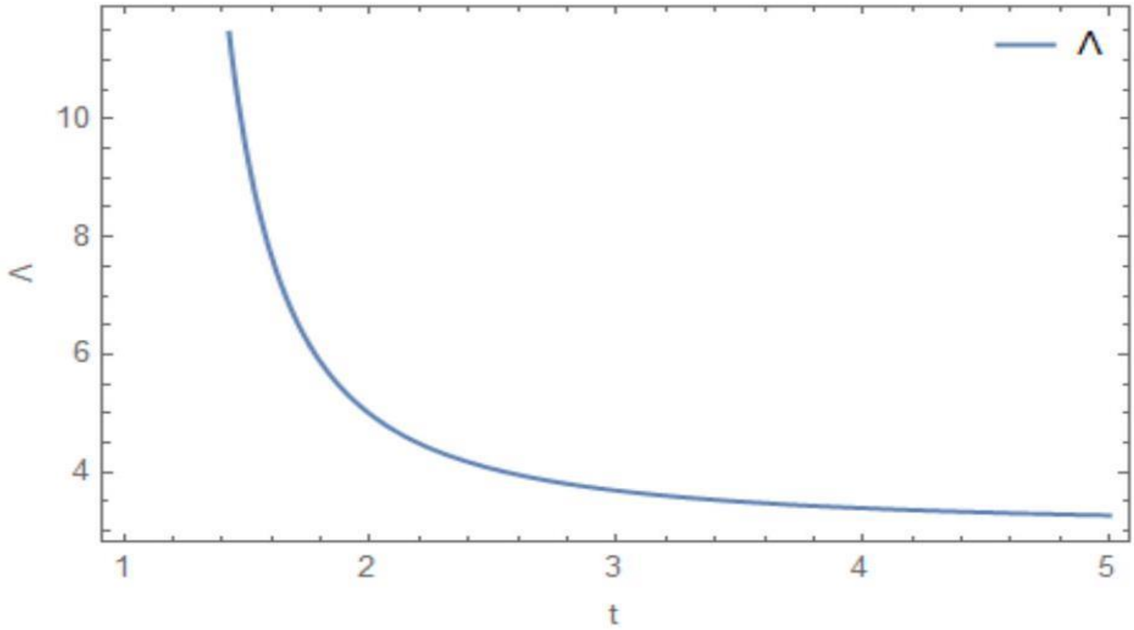

Figure 5: Case (i) $\rightarrow \infty$: The relation between Cosmological Constant $\Lambda(t)$ and Cosmic time $t,\ m = 3,\ \alpha = 1,\ n = 3,\ \Lambda_0 = 3,\ \Lambda_1 = 1, \Lambda_2 = 2,\ t_0 = 1.$

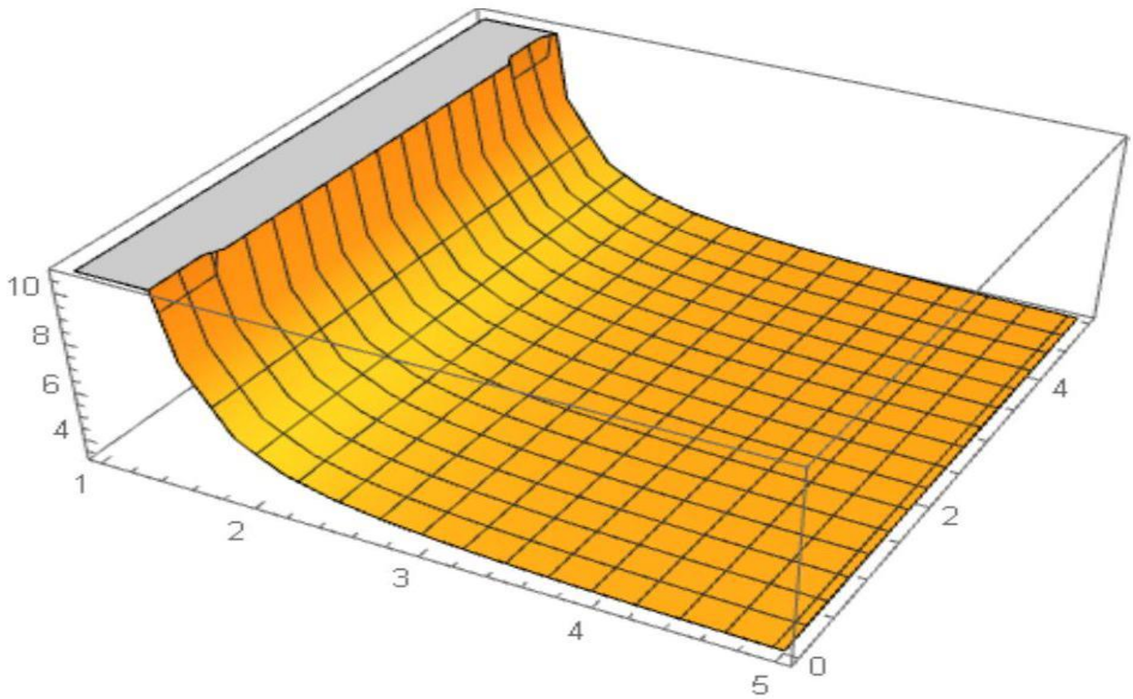

Figure 6: Case (i) $\rightarrow \infty$: The relation between Cosmological Constant $\Lambda(t)$ and Cosmic time t in 3-Dimensional view, $m = 3,\ \alpha = 1,\ n = 3, \Lambda_0 = 3,\ \Lambda_1 = 1,\ \Lambda_2 = 2,\ t_0 = 1$

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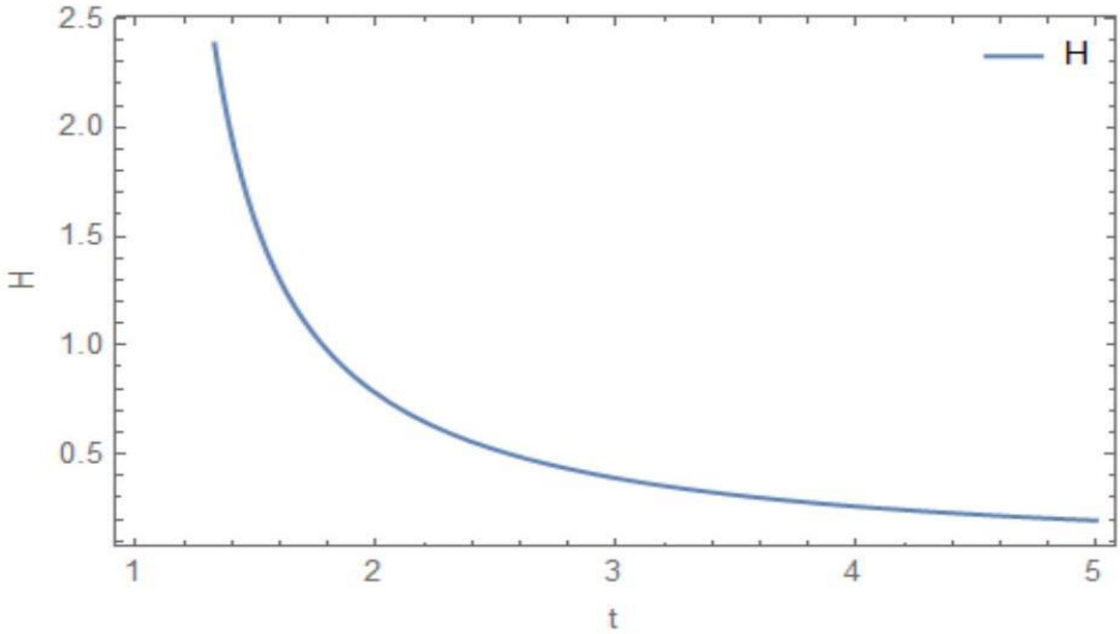

Figure 7: Case (i) $\rightarrow \infty$: The relation between Hubble parameter H and Cosmic time t, $m = 3$, $\alpha = 1$, $n = 3$, $\Lambda_0 = 3$, $\Lambda_1 = 1$, $\Lambda_2 = 2$, $t_0 = 1$.

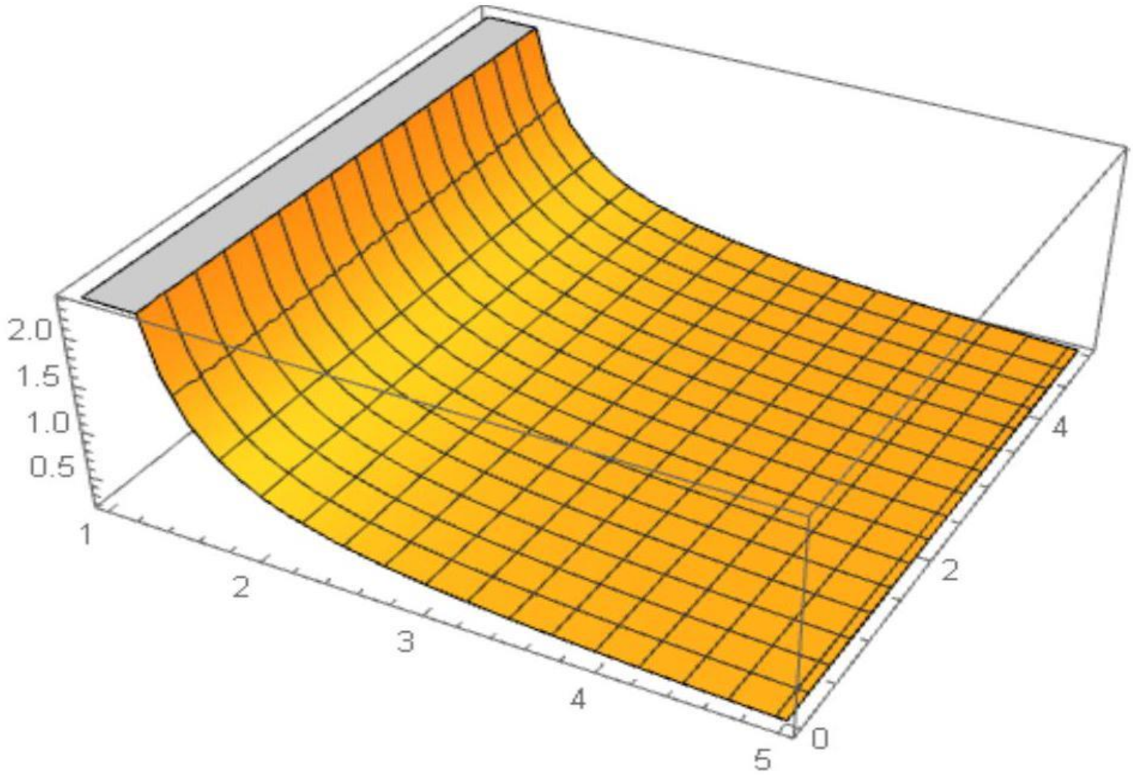

Figure 8: Case (i) $\rightarrow \infty$: The relation between Hubble parameter H and Cosmic time t in 3-Dimensional view, $m = 3$, $\alpha = 1$, $n = 3$, $\Lambda_0 = 3$, $\Lambda_1 = 1, \Lambda_2 = 2$, $t_0 = 1$.

- Based on the figure (11), it is evident that the energy density ρ decreases as time t increases within the range $1 < n < 4$. Furthermore, based on Equation (45), it is

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determined that ρ must be greater than or equal to zero, and this requirement influences the selection of constant values.

- As the variable t increases, both the Hubble parameter H and the expansion scalar Θ similarly increase.
- As the variable t approaches zero, it is noted that the value of λ remains constant.
- It is evident that the spatial volume is zero at $t = 0$ for appropriate values of constants, and it grows as t increases.
- Since, $\lim_{t\to\infty} \frac{\sigma}{\Theta} =$ constant, the model does not approach isotropy at late time.
- When the constants are appropriately chosen, we obtain a result of $q < 0$, which supports the preference for accelerating models. Therefore, we can conclude that the outcome aligns with the present findings of SNe Ia and CMBR.
- The behavior of the mean anisotropy parameter is contingent upon the value of n.
- As $n \to 1, q \to 0$ and A_h vanishes.

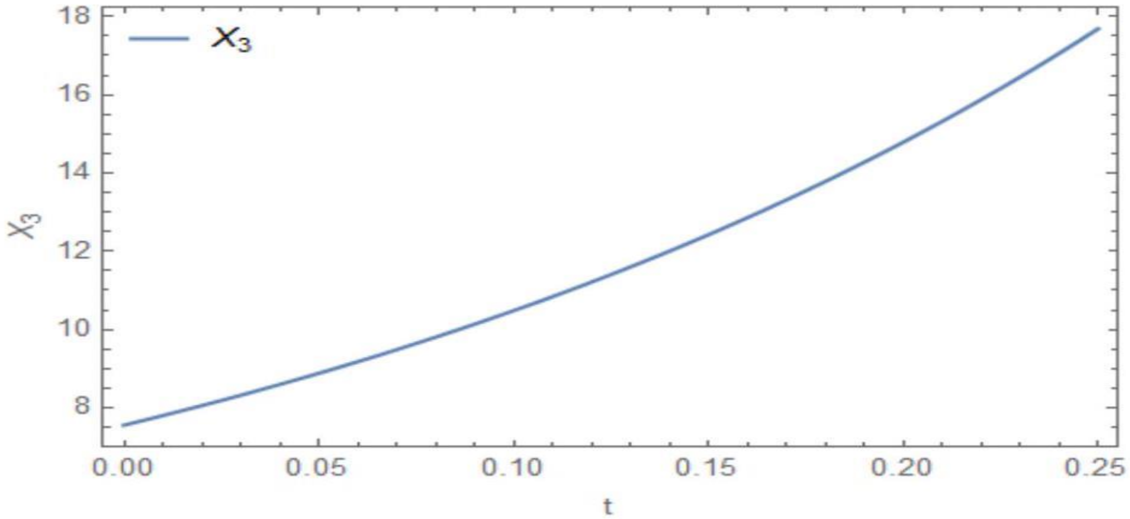

Figure 9: Case (ii) $\to 0$: The relation between X_3 and time t, $m = 3$, $\alpha = 1$, $n = 3$, $\Lambda_0 = -3$, $\Lambda_1 = 1$, $\Lambda_2 = 0.05$, $d_1 = 1$, $d_2 = 1$, $Z_1 = 1.36$, $Z_2 = 1.16, Z_3 = -1.5$, $Z_4 = 0.9024, t_0 = 1$.

5. Conclusion:

This paper examines the modified equation of state (EoS) of the Takabayasi string, which is represented by the equation $\rho = (1 + \omega)\lambda + \Lambda(t)$. The investigation focuses on the existence of a bulk viscous fluid and the solution of

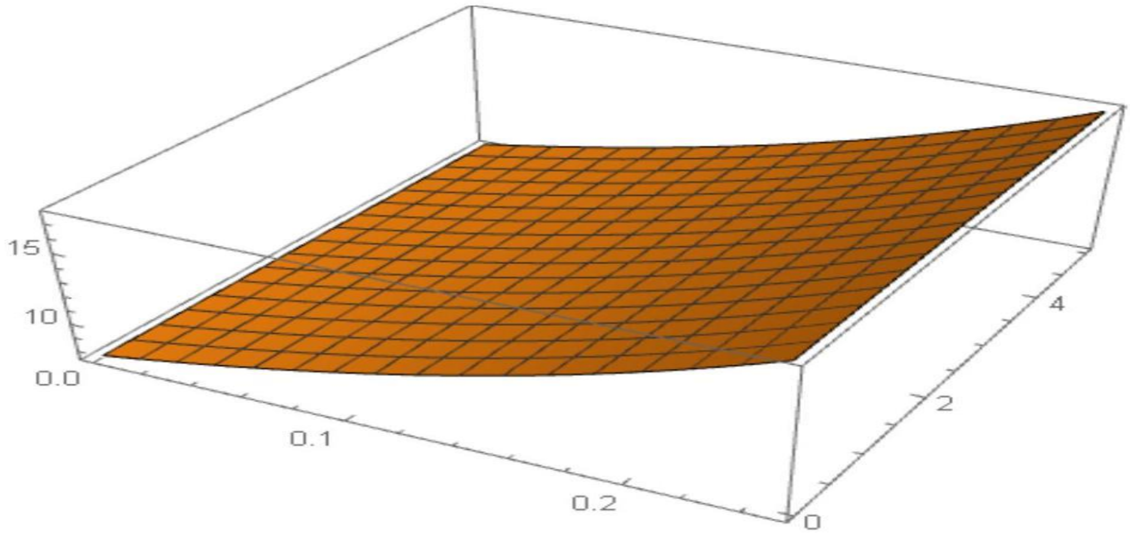

Figure 10: Case (ii) → 0 : The relation between X_3 and Cosmic time t in 3-Dimensional view, $m = 3,\ \alpha = 1,\ n = 3,\ \Lambda_0 = -3,\ \Lambda_1 = 1,\quad \Lambda_2 = 0.05,\ d_1 = 1,\ d_2 = 1,\ Z_1 = 1.36,\ Z_2 = 1.16,\ Z_3 = -1.5,\ Z_4 = 0.9024, t_0 = 1.$

the field equation within the Bianchi III model. We have examined the two intriguing scenarios based on the value of ω, specifically the extreme situations of ω approaching infinity and ω approaching zero. It is observed that when the angular frequency ω approaches infinity, we solved Equation (26) to find the value of the scale factor C and determined the different physical parameters. Similarly, when ω approaches zero, we solved the differential equation (43) to find the value of C and achieved the precise solution. We next thoroughly examined all the physical parameters by graphing them.

Case (i): $\omega \to \infty$

From graph (1), it is evident that as t grows, the scale factors increase, but the energy density ρ, cosmological term Λ, and Hubble parameter H decrease, as shown in figures (3), (5), and (7). The energy density criterion, where ρ

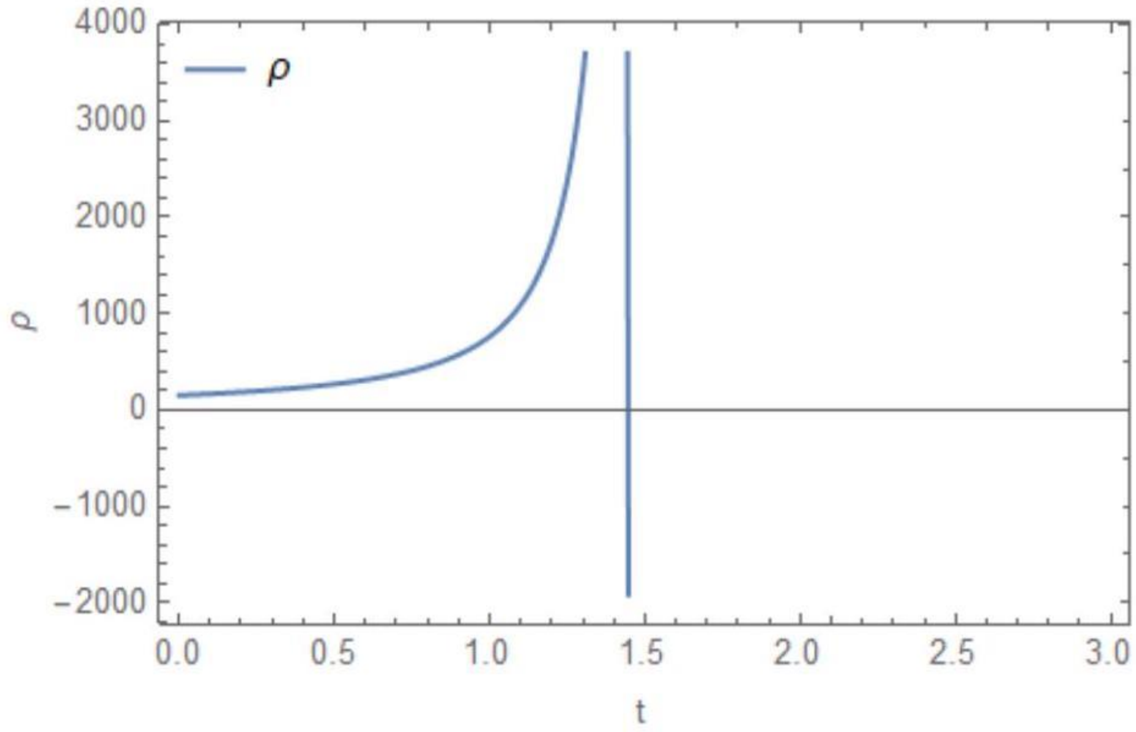

Figure 11: Case (ii) → 0 : The relation between energy density ρ and Cosmic time t, $m = 3$, $\alpha = 1$, $n = 3$, $\Lambda_0 = -3$, $\Lambda_1 = 1$, $\Lambda_2 = 0.05$, $d_1 = 1$, $d_2 = 1$, $Z_1 = 1.36$, $Z_2 = 1.16$, $Z_3 = -1.5$, $Z_4 = 0.9024$, $t_0 = 1$. is greater than zero, is fulfilled. From equation (34), it is evident that the value of lambda is zero. It has been noted that the cosmological constant Λ is positive and diminishes as time t grows.

Case (ii): $\omega \to 0$, here we discuss the modified geometric string. The energy density ρ is a decreasing function of time t for the range $1 < n < 4$, as can be seen in figure (11), which makes this assertion very evident. In addition, the values of constants are selected based on the equation (46), where rho is equal to zero. The Hubble parameter H and the expansion scalar θ both rise in proportion to the increase in t. As the value of t approaches zero, it is noted that the value of λ remains unchanged. As $t = 0$ for proper values of constants, it is possible to observe that the spatial volume is zero, and that it increases as the value of t increases during the process. Given that $\lim_{t\to\infty} \frac{\sigma}{\theta} =$ constant, it may be concluded that the model does not approach isotropy at relatively late times. When the constants are determined to be suitable, we obtain a value of $q < 0$, which is favorable for accelerating

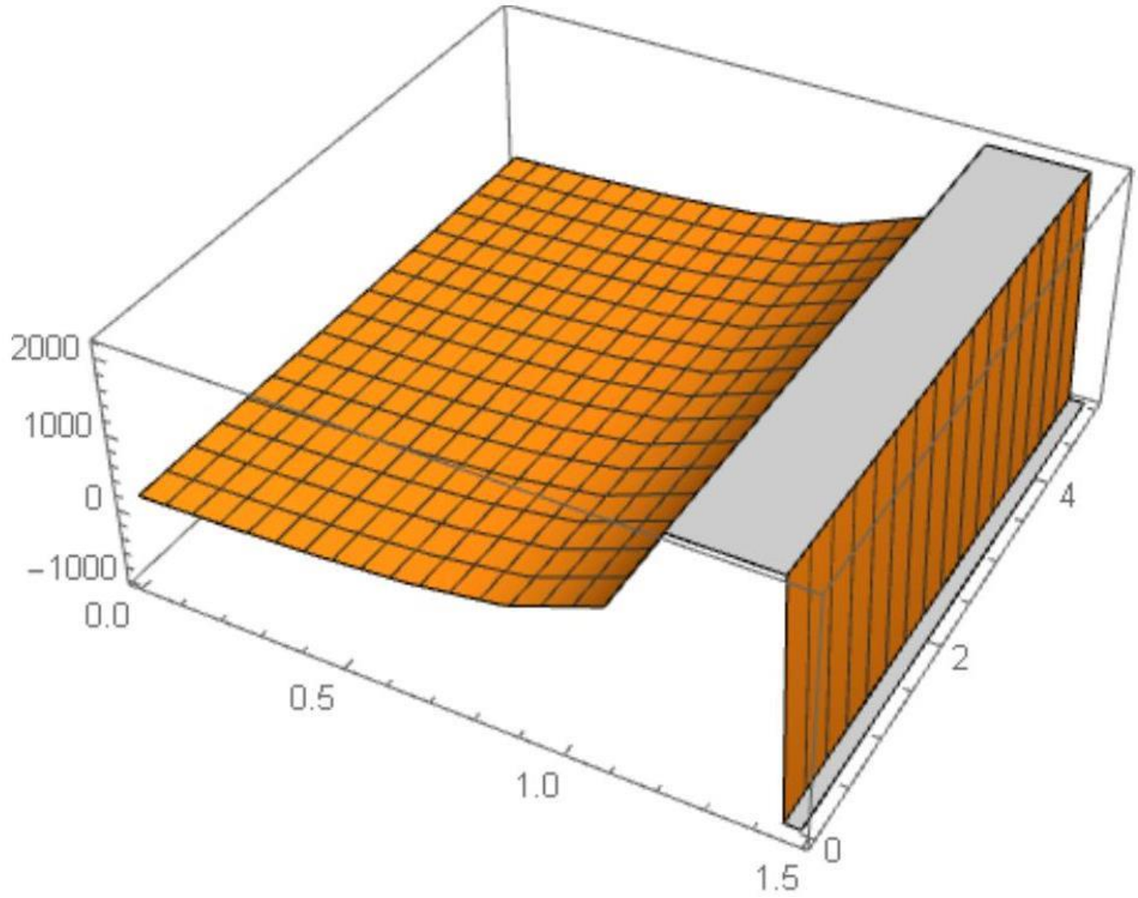

Figure 12: Case (ii) → 0 : The relation between energy density ρ and Cosmic time t in 3-Dimensional, $m = 3,\ \alpha = 1,\ n = 3,\ \Lambda_0 = -3,\ \Lambda_1 = 1,\ \Lambda_2 = 0.05,\ d_1 = 1,\ d_2 = 1,\ Z_1 = 1.36,\ Z_2 = 1.16,\ Z_3 = -1.5,\ Z_4 = 0.9024, t_0 = 1$.

models. The conclusion that we can draw from this is that the result is in agreement with the most recent measurements of SNe Ia and CMBR. It is dependent on the value of n that the dynamics of the mean anisotropy parameter are determined. As n, q, and A_h disappear, the value of n is considered to be 1 .

The model is experiencing acceleration for values of n greater than 1 . Therefore, the anisotropy in the models is consistently preserved in both models. Furthermore, the model exhibiting a negative deceleration parameter aligns with the latest Supernovae Ia findings, indicating that the universe is currently experiencing acceleration during its later stages. Thus, the model accurately characterizes a universe that is always growing, undergoing shear, and without rotation. Current findings and literature support the speeding

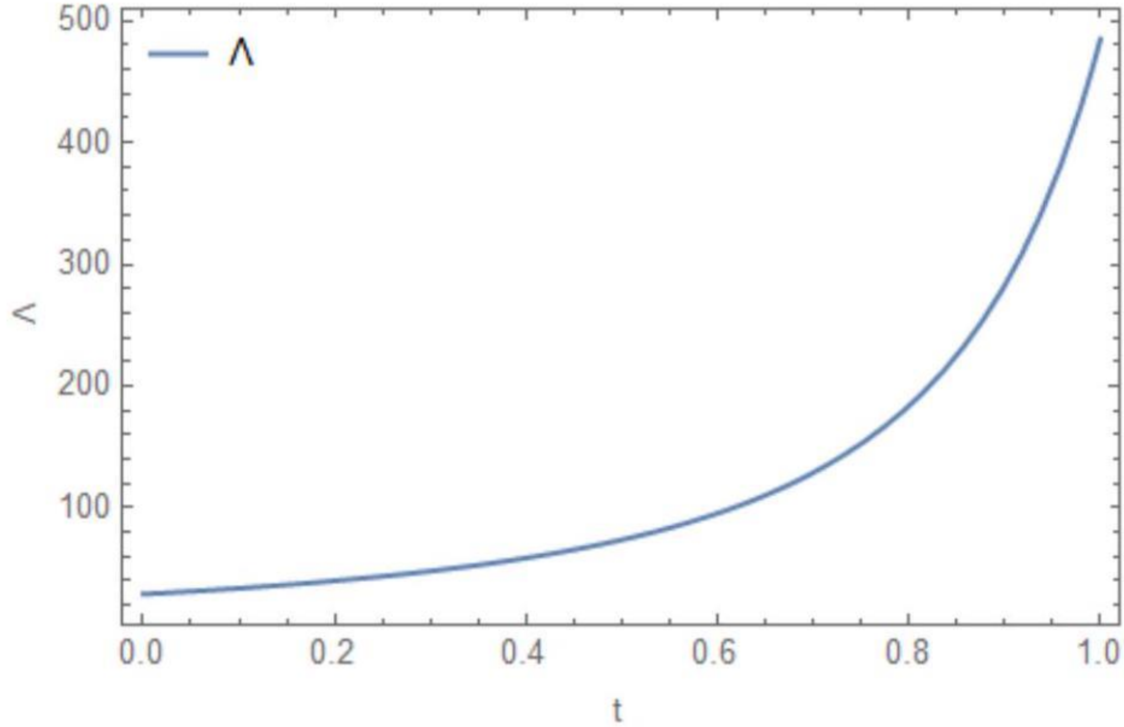

Figure 13: Case (ii) → 0 : The relation between Cosmological Constant $\Lambda(t)$ and Cosmic time t, $m = 3$, $\alpha = 1$, $n = 3$, $\Lambda_0 = -3$, $\Lambda_1 = 1$, $\Lambda_2 = 0.05$, $d_1 = 1$, $d_2 = 1$, $Z_1 = 1.36$, $Z_2 = 1.16$, $Z_3 = -1.5$, $Z_4 = 0.9024$, $t_0 = 1$.

hypothesis of the cosmos.

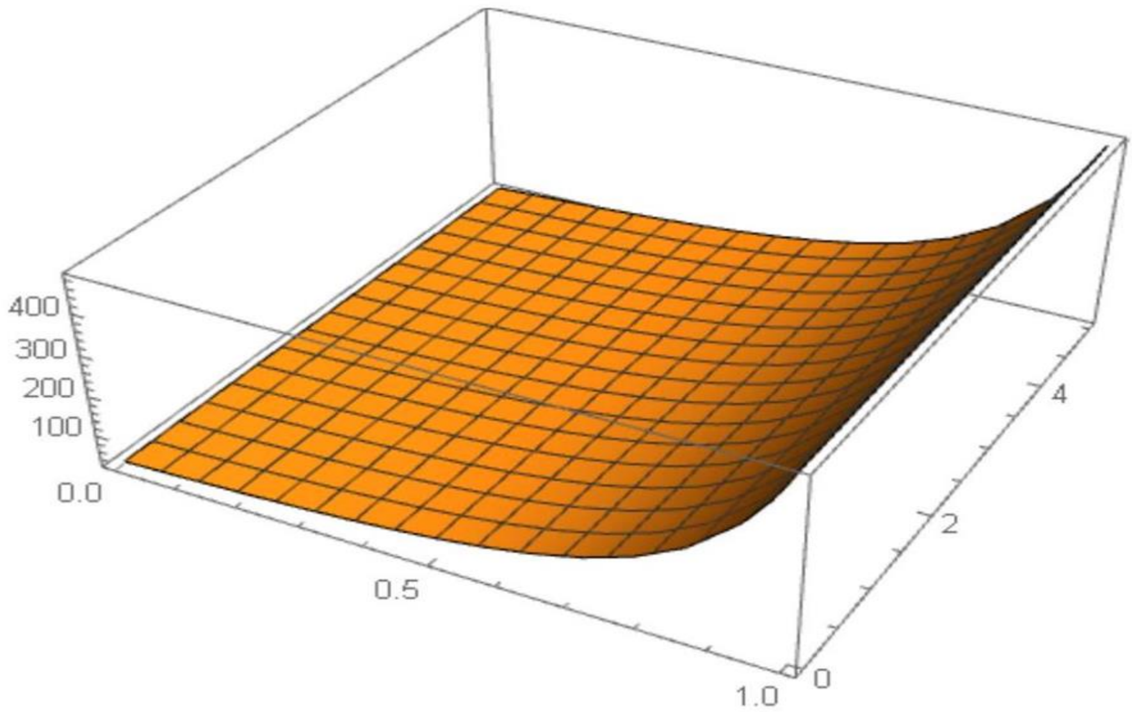

Figure 14: Case (ii) → 0 : The relation between Cosmological Constant $\Lambda(t)$ and Cosmic time t in 3-Dimensional view, $m = 3$, $\alpha = 1$, $n = 3$, $\Lambda_0 = -3$, $\Lambda_1 = 1$, $\Lambda_2 = 0.05$, $d_1 =$

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1, $d_2 = 1$, $Z_1 = 1.36$, $Z_2 = 1.16$, $Z_3 = -1.5$, $Z_4 = 0.9024$, $t_0 = 1$.

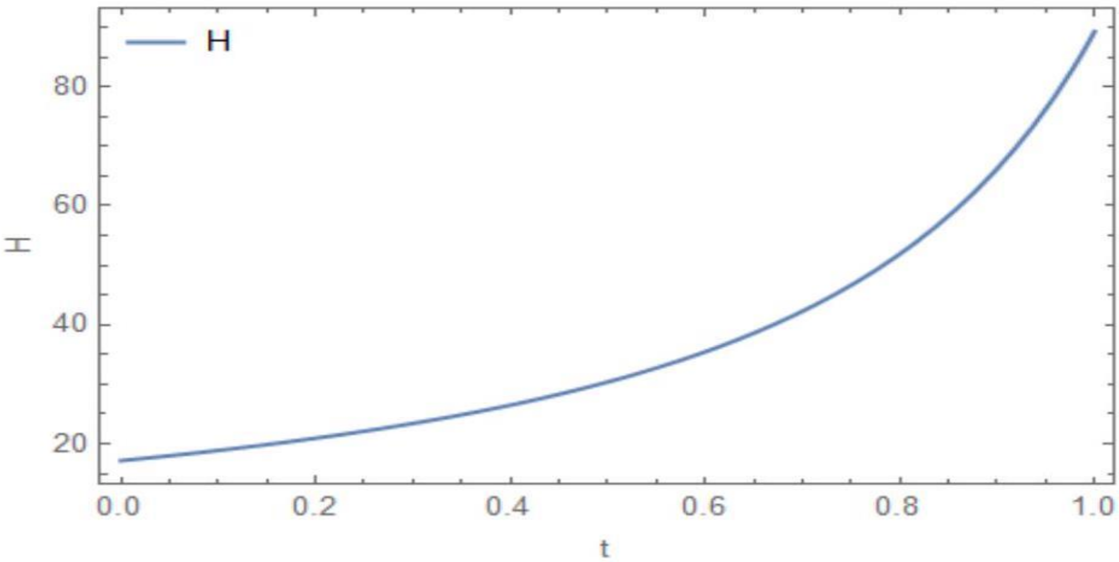

Figure 15: Case (ii) → 0 : The relation between Hubble parameter H and Cosmic time t, $m = 3$, $\alpha = 1$, $n = 3$, $\Lambda_0 = -3$, $\Lambda_1 = 1$, $\Lambda_2 = 0.05$, $d_1 = 1$, $d_2 = 1$, $Z_1 = 1.36$, $Z_2 = 1.16$, $Z_3 = -1.5$, $Z_4 = 0.9024$, $t_0 = 1$.

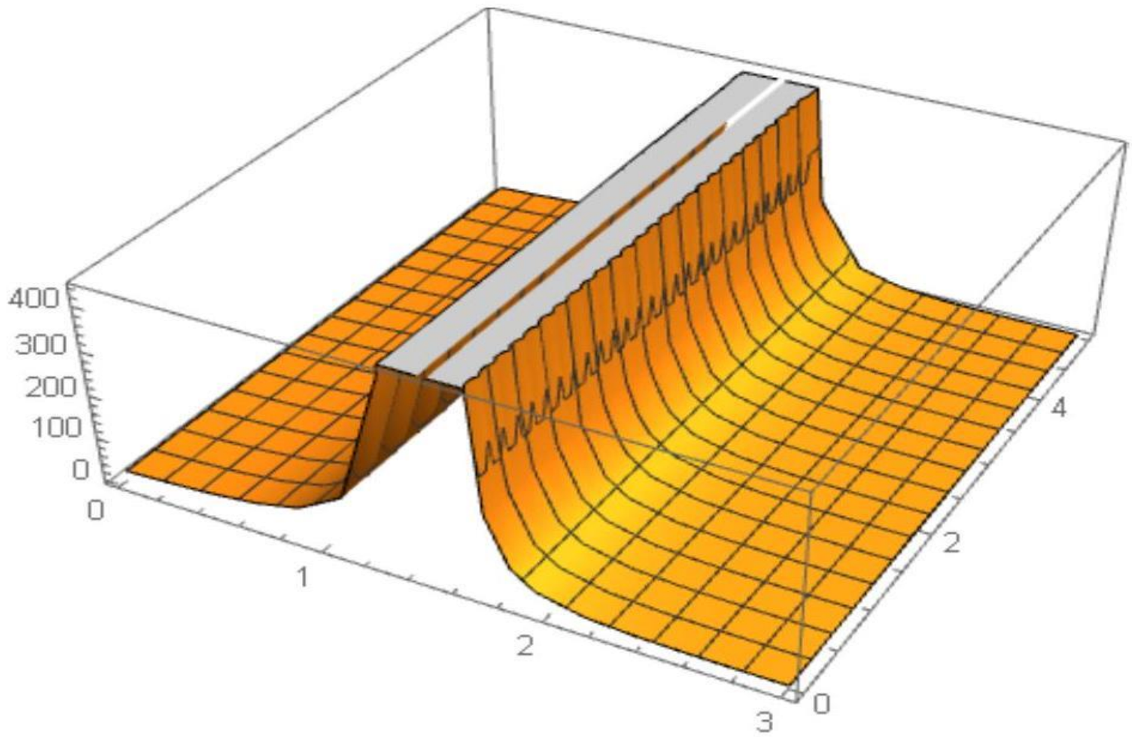

Figure 16: Case (ii) → 0 : The relation between Hubble parameter H and Cosmic time t in 3-Dimensional view, $m = 3$, $\alpha = 1$, $n = 3$, $\Lambda_0 = 3$, $\Lambda_1 = 1, \Lambda_2 = 2$, $t_0 = 1$ of a time-dependent Λ term Phys. Rev. D, Vol. 46, 2404 (1992).

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