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An Integrated Deteriorating Inventory Model with Imperfection Inspection, Variable Demand, and Trade-Credit under Inflation

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Abstract

This study develops an inventory model addressing the complexities of deteriorating items under an imperfection inspection policy, variable demand influenced by time and selling price, and trade-credit financing in the presence of inflation. Imperfections in inventory are identified through a comprehensive inspection process, minimizing the impact of defective items on overall operations. Demand is assumed to vary dynamically, reflecting dependencies on temporal factors and pricing strategies. The trade-credit policy, allowing for delayed payments, introduces additional flexibility for retailers while influencing inventory decisions. Inflation is incorporated to reflect its impact on cost structures and purchasing power, further enriching the model's practical relevance. Through mathematical formulations and numerical analyses, the model provides actionable insights into optimizing inventory policies, balancing the costs of deterioration, inspection, and trade-credit, and adapting to market conditions. The findings offer valuable strategies for businesses managing deteriorating inventories in fluctuating economic environments.

Keywords- trade credit, imperfect quality items, inventory model, production model

Introduction and literature review

In today's competitive and dynamic market environment, inventory management is a critical factor in ensuring business success. Effective inventory policies not only optimize resource utilization but also address the challenges posed by deterioration, demand variability, and financial constraints. Deteriorating items, such as perishable goods, chemicals, or pharmaceuticals, lose value over time, necessitating specialized management strategies to minimize losses. Additionally, demand for such items often depends on factors like selling price and time, introducing further complexities into inventory decision-making. Trade-credit policies, where suppliers offer delayed payment terms to retailers, have become a common practice to foster business relationships and enhance cash flow. This credit arrangement impacts inventory policies by affecting the timing and volume of orders. Moreover, inflation the persistent increase in price levels affects costs, demand, and revenue, necessitating the incorporation of inflationary effects into inventory models. An often-overlooked aspect in inventory management is the role of inspection policies, especially for deteriorating items. Imperfect items, if undetected, can lead to reduced customer satisfaction, increased returns, and diminished profitability. Thus, a rigorous inspection policy is essential to identify and segregate imperfect items before they impact operations.

This study integrates these elements to develop an inventory model that addresses deterioration, variable demand influenced by time and price, inspection policies, trade-credit arrangements, and inflation. The model aims to provide insights into managing inventories efficiently in a complex and realistic business environment. The study of deteriorating items began with the seminal work of Ghare and Schrader (1963), who introduced exponential decay to model deterioration. Classical inventory models often assumed constant demand, but real-world

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demand is influenced by time, price, and promotional efforts. Researchers like Nita H. Shah et al. (2006) have explored demand functions that depend on time and selling price, offering a more realistic approach. The concept of trade-credit was first incorporated into inventory models by Goyal (1985), who analysed the effects of permissible delay in payments on the economic order quantity. Later, Teng (2002) extended this work by considering trade-credit terms offered to both retailers and customers. The inclusion of trade-credit terms has since become a standard practice in inventory models, addressing the financial realities faced by businesses. Inspection policies are crucial for detecting imperfect items, especially in deteriorating inventory systems. Salameh and Jaber (2000) introduced a model that considers inspection and quality assurance in imperfect production systems. Min et. al. (2010) a retailer who purchases the items enjoys a fixed credit period offered by his/her supplier and, in turn, also offers a credit period to his/her customers in order to promote the market competition.Teng et. al. (2023) extended the constant demand to a linear non-decreasing demand function of time and incorporate a permissible delay in payment under two levels of trade credit into the model. The supplier offers a permissible delay linked to order quantity, and the retailer also provides a downstream trade credit period to its customers. Sarkar et al. (2015) assume that the suppliers offer full trade-credit to retailers but retailers offer partial trade-credit to their customers. Building on this, researchers have explored strategies to minimize inspection costs and improve defect detection rates, emphasizing the role of inspection in ensuring product quality. Singh and Singh (2017) considered an inventory model with demand depended on selling price and stock. Here they also consider trade credit policy. Tayal et al. (2021) created an integrated inventory model with demand depend on selling price and stock and also, consider holding cost as variable. here they show how inflation rate is affecting this inventory model. Handa et. al. (2021) developed an economic order quantity model in which their demand is depend on stock and price dependent and also, they consider effect of trade credit policy with partial backlogging. Padiyar et. al. (2022) considered a supply chain model in which they highlighted deteriorating product with inflation. Handa et. al. (2024) developed a reverse logistics inventory model in which they consider imperfect production with partial backlogging in supply chain but here the concept of inspection is missing. While extensive research exists on individual aspects such as deterioration, demand variability, trade-credit, and inflation, few studies integrate these factors into a unified framework. Moreover, the role of inspection policies in managing imperfections within deteriorating inventory systems remains underexplored, especially in the context of variable demand and trade-credit. This study addresses these gaps by developing a comprehensive model that incorporates all these elements, offering practical insights for businesses operating in complex environments.

Assumptions and Notations

The following assumptions are taken in this model-

- The lead time is negligible.
- Inflation is considered.
- Shortage is allowed which is partial backlogged.
- Demand is considered as function of selling price and time that is $D(p,t) = a - bp + ct$. Where a and b are positive number.
- Deterioration rate is considered as function of time $\theta(t) = \theta t$.

- The retailer follows a capital investment, $H(\omega)$, for improving the item quality for reducing the defective items, which is given as $H(\omega) = \frac{1}{\lambda_0} ln \frac{\omega_u}{\omega}$ where $0 \leq \omega \leq \omega_u$. (This function was first used by Hall, and is being widely used by many researchers like Porteus, Ouyang et al.) Here, ω_u is proportion of defective items before improving the production process and λ_0 is percentage decrease in ω, per increase in $H(\omega)$.
- Errors during inspection may be made by the store. Our study addresses two sorts of faults. Type I errors arise when a store accepts damaged products as non-defective by mistake. Type II occurs when a store erroneously rejects non-faulty products as defective.
- If $T > M$, the retailer settles the account at time M and pays interest charges on in-stock products at rate $I_c$throughout the period $[M, T]$. If $T < M$, the store adjusts the account at time M and does not charge interest on stock during the cycle.

The following notations are taken in this model-

Parameters	Descriptions
a	Scaling factor
b	Scaling factor
c	Scaling factor
O	Ordering cost
y	size of each shipment from supplier to retailer
C_{h1}	Holding cost for defective items
ω	proportion of defective items
z	proportion of defective items that can be reworked.
s	Purchasing cost
r	Inflation rate
t_1	Time where production stopped
t_2	Time where inventory become zero
t_3	Time the production is restarted to recover both demand and backordered items until the inventory level reaches the value 0
t_4	Time where inventory become zero.
θ	Deteriorating rate
C_{h2}	Holding cost for non-defective items
C_d	Deterioration cost
S_c	Shortage cost
S_l	Lost sale cost
C_0	unit opportunity cost of capital investment per unit per unit time,
C_p	unit inspection cost per items per unit time
C_{e1}	unit cost of falsely rejecting non-defective items per unit time
C_{e2}	unit cost of falsely accepting defective items per unit time
τ	proportion of defective items that can be reworked
γ	Backorder rate
M	Trade credit period of retailer
λ_0	rate of percentage decrease in ω per increase in $H(\omega)$
I_e	Interest earned
I_c	Interest charged
p	Selling price
T	Total cycle length

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Mathematical model-

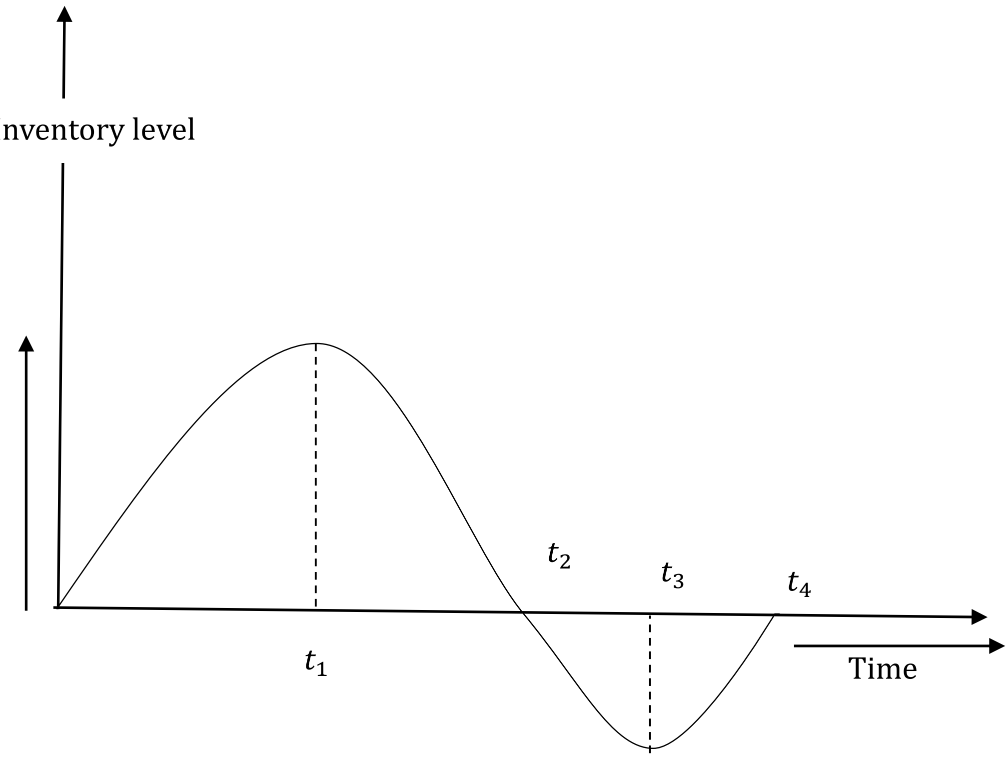

Fig:1- showing graphical representation of inventory level for this model

Based on our assumptions, initially inventory at $t = 0$, and increase due to production till $t = t_1$ after that deplete due to demand and deterioration. And after that shortage is occur till $t = t_3$ and here from $t = t_3$ again fulfil the demand and backorder quantity till $t = t_4$. The differential equations of proposed model are written as-

$$\frac{dI_1(t)}{dt} + \theta(t)I_1(t) = z(1-\omega) - D(t,p), t \in [0, t_1) \quad (1)$$

With the initial condition $I(0) = 0$

$$\frac{I_2(t)}{dt} + \theta(t)I_2(t) = -D(t,p), t \in [t_1, t_2) \quad (2)$$

With the boundary condition $I_2(t_2) = 0$

$$\frac{I_3(t)}{dt} = -\gamma D(t,p), t \in [t_2, t_3) \quad (3)$$

With the boundary condition $I_3(t_2) = 0$

$$\frac{I_4(t)}{dt} = z(1-\omega) - D(t,p), t \in [t_3, t_4) \quad (4)$$

With the boundary condition $I_4(t_4) = 0$

The solution of these equation with conditions are

$I_1(t)=\frac{z(1-\omega)-(a-bp)}{\theta}\left(1-e^{-\theta t}\right)-\frac{ct}{\theta}+\frac{c}{\theta^2}\left(e^{-\theta t}-1\right)$, $I_2(t)=\frac{c}{\theta^2}\left(1-e^{\theta(t_2-t)}\right)-\frac{a-bp+ct}{\theta}-\frac{a-bp+ct_2}{\theta}e^{\theta(t_2-t)}$, $I_3(t)=\gamma(a-bp)(t_2-t)+\frac{\gamma c}{2}(t_2^2-t^2)$, $I_4(t)=\left(z(1-\omega)-(a-bp)\right)(t-t_4)+\frac{c}{2}(t_4^2-t^2)$ (5)

Using the condition $I_1(t_1)=I_2(t_1)$, we get

$$t_1=\frac{1}{2}\left(\frac{cT}{\theta}-\frac{c}{\theta^2}-\frac{a-bp}{\theta}\right)e^{\theta T}+\frac{1}{2}\left(\frac{z(1-\omega)}{\theta}-\frac{c}{\theta^2}-\frac{(a-bp)(T+1)}{\theta}\right)-2ce^{2\theta T}\left(\frac{c(T+1)}{\theta^2}-\frac{(a-bp)T}{\theta}\right) \quad (6)$$

Again using $I_2(t_2)=I_3(t_2)$, then we get,

$$t_2=\left(\frac{c}{\theta^2}+\frac{cT}{\theta}+\frac{a-bp}{\theta}\right)e^{\theta(T-t_1)}-\frac{\theta\left(\gamma(a-bp)T+\frac{\gamma cT^2}{2}\right)}{2cT+(a-bp)} \quad (7)$$

For continuity condition $I_3(t_3)=I_4(t_3)$ again we get

$$t_3=\left(\frac{T^2\gamma c}{2}+\frac{ct_2}{2}\right)^2+T\left(\gamma(a-bp)+z(1-\omega)-(a-bp)\right) \quad (8)$$

Using the condition $I_4(t_4)=0$, we get

$$t_4=\frac{z(1-\omega)-(a-bp)}{c}+\frac{(cT^2-2T)\sqrt{z(1-\omega)-(a-bp)^2}}{c} \quad (9)$$

Now, find all inventory cost for this inventory model-

1. Ordering cost- $OC=yO$ (10)
2. Holding cost for defective items is calculated as-

$$HCD=C_{h_1}y\omega\left[\int_0^{t_1}I_1(t)e^{-rt}dt+\int_{t_1}^{t_2}I_2(t)e^{-rt}\,dt\right]$$

$$=C_{h_1}y\omega\left[\left(\frac{z(1-\omega)-(a-bp)}{\theta}\right)\left(\frac{e^{-rt_2}-1}{-r}-\frac{1}{\theta+r}\left(e^{-(\theta+r)t_2}-1\right)\right)-\frac{c}{\theta}\left(\frac{t_2e^{-rt_2}}{-r}-\frac{e^{-rt_2}-1}{r^2}\right)\right] \quad (11)$$

3. Holding cost for non-defective items is calculated as-

$$HCND=C_{h_2}y(1-\omega)\left[\int_0^{t_1}I_1(t)e^{-rt}dt+\int_{t_1}^{t_2}I_2(t)\,e^{-rt}dt\right]$$

$$=C_{h_2}y(1-\omega)\left[\left(\frac{z(1-\omega)-(a-bp)}{\theta}\right)\left(\frac{e^{-rt_2}-1}{-r}-\frac{1}{\theta+r}\left(e^{-(\theta+r)t_2}-1\right)\right)-\frac{c}{\theta}\left(\frac{t_2e^{-rt_2}}{-r}-\frac{e^{-rt_2}-1}{r^2}\right)\right] \quad (12)$$

4. The deteriorating cost is

$$DC=C_d\theta\left[\int_0^{t_1}(z-(a-bp+ct))e^{-rt}dt+\int_{t_1}^{t_2}I_2(t)\,e^{-rt}dt\right]$$

$$=C_d\theta\left[(z-(a-bp))t_1+\frac{ct_1^2}{2}+\left(\frac{z(1-\omega)-(a-bp)}{\theta}\right)\left(\frac{e^{-rt_2}-e^{-rt_1}}{-r}-\frac{e^{-(\theta+r)t_2}-e^{-(\theta+r)t_1}}{\theta+r}\right)-\frac{c}{\theta}\left(\frac{t_2e^{-rt_2}-t_1e^{-rt_1}}{-r}-\frac{e^{-rt_2}-e^{-rt_1}}{r^2}\right)\right] \quad (13)$$

5. Shortage cost is calculating for interval $[t_2,t_3]$

$$SC = yS_c\gamma q\int_{t_2}^{t_3}(a-bp+ct)\,e^{-rt}dt = yS_c\gamma\left[(a-bp)(e^{-rt_3}-e^{-rt_2}) - c\left(\frac{t_3e^{-rt_3}-t_2e^{-rt_2}}{r}+\frac{e^{-rt_3}-e^{-rt_2}}{r^2}\right)\right] \quad (14)$$

6. Lost sale cost for interval $[t_3, t_4]$

$$LSC = (1-\gamma)yS_l\int_{t_3}^{t_4}I_4(t)\,dt = (1-\gamma)yS_l\left[\left(z(1-\omega)-(a-bp)\right)\left(\left(\frac{t_4e^{-rt_4}-t_3e^{-rt_3}}{-r} - \frac{e^{-rt_4}-e^{-rt_3}}{r^2}\right)+\frac{t_4(e^{-rt_4}-e^{-rt_3})}{r}\right)+\frac{c}{2}\left(t_4^2\frac{e^{-rt_4}-e^{-rt_3}}{-r} - (\frac{t_4^2e^{-rt_4}-t_3^2e^{-rt_3}}{-r} - \frac{2}{r^2}(t_4e^{-rt_4} - t_3e^{-rt_3}) - \frac{2}{r^3}(e^{-rt_4}-e^{-rt_3}))\right)\right] \quad (15)$$

7. Opportunity cost-

$$OPC = C_oH(\omega)Q = \frac{C_o}{\lambda_0}ln\frac{\omega_u}{\omega}\,zy(1-\omega) \quad (16)$$

8. Annual inspection cost

$$AIC = yC_pQ = y^2\,C_pz(1-\omega) \quad (17)$$

9. Annual cost due to type-1 error is -

$$ACE(1) = \frac{D(t,p)C_{e1}\omega(1-\tau)}{T(1-\omega\tau)} = \frac{(a-bp+ct_3)C_{e1}\omega(1-\tau)}{T(1-\omega\tau)} \quad (18)$$

10. Annual cost due to type-2 error is-

$$ACE(2) = \frac{D(t,p)\omega\tau C_{e_2}}{T(1-\omega\tau)} = \frac{(a-bp+ct_2)\omega\tau C_{e2}}{T(1-\omega\tau)} \quad (19)$$

11. The annual penalty cost for retailer to the customer is calculated as

$$PC = C_p\left[\frac{(a-bp+ct_3)C_{e1}\omega(1-\tau)}{T(1-\omega\tau)}+\frac{(a-bp+ct_2)\omega\tau C_{e2}}{T(1-\omega\tau)}\right] \quad (20)$$

Now, find interest earn and interest charge for all cases-

Here are many cases arise in the inventory model but we are focus on mainly two cases when $0 \le M \le T$ and $0 \le T \le M$.

Case-1 when $0 \le M \le T$

12. Interest earned

$$IE = I_eyp\int_0^M t\,D(t,p)\,e^{-rt}dt = I_eyp\left[(a-bp)\left(\frac{Me^{-rt}}{-r}-\frac{e^{-rM}-1}{r^2}\right)+c\left(\frac{M^2e^{-rM}}{-r}-\frac{2Me^{-rM}}{r^2}-\frac{2(e^{-rM}-1)}{r^3}\right)\right] \quad (21)$$

13. Interest charged

$$IC = syI_c\int_M^T I_2(t)\, e^{-rt}dt = syI_c\left[\frac{c}{\theta^2}\left(\frac{e^{-rT}-e^{-rM}}{-r}+\frac{e^{\theta(t_2-T)-rT}-e^{\theta(t_2-M)-rM}}{\theta+r}\right)+\left(\frac{a-bp}{r\theta}\right)(e^{-rT}-e^{-rM})-c\left(\frac{Te^{-rT}-Me^{-rM}}{-r}-\frac{e^{-rT}-e^{-rM}}{r^2}\right)+\left(\frac{a-bp+ct_2}{\theta}\right)\left(\frac{e^{\theta(t_2-T)-rT}-e^{\theta(t_2-M)-rM}}{\theta+r}\right)\right] \quad (22)$$

Case-2 when $0 \le T \le M$

14. Interest earned

$$IE = pI_e y\left[\int_0^T(a-bp+ct)te^{-rt}dt+(M-T)\int_0^T(a-bp+ct)e^{-rt}dt\right]$$

$$= pyI_e\left[(a-bp)\left(\frac{Te^{-rT}}{-r}-\frac{e^{-rT}-1}{r^2}\right)+c\left(\frac{T^2e^{-rT}}{-r}-\frac{2Te^{-rT}}{r^2}-\frac{2(e^{-rT}-1)}{r^3}\right)+(M-T)\left((a-bp)\frac{e^{-rT}}{-r}+c\left(\frac{Te^{-rT}}{-r}-\frac{e^{-rT}-1}{r^2}\right)\right)\right] \quad (23)$$

15. Interest charged- Here, trade credit period is greater than total cycle length so that retailer has not charged any kind of interest by supplier so that $IC = 0$.

From eq. (10) to (23) we have all cost. Now put in eq. (24) then we get total cost for both cases. The total annual cost is

$$TC = \frac{1}{T}[OC + HCD + HCND + DC + SC + LSC + OPC + AIC + PC + IC - IE] \quad (24)$$

Solution methodology-

For check the proposed total cost for this inventory model is convex consider following steps-

Step 1.- First of all find first derivative of $TC\{p,T\}$ with respect to p and T i.e. $\frac{\partial TC\{p,T\}}{\partial p}$ and $\frac{\partial TC\{p,T\}}{\partial T}$.

Step 2.- Put both equation from step-1 equal to zero and get optimal points after that find double derivative of $TC\{p,T\}$ i.e $\frac{\partial^2 TC\{p,T\}}{\partial p^2}, \frac{\partial^2 TC\{p,T\}}{\partial p\partial T}, \frac{\partial^2 TC\{p,T\}}{\partial T\partial p}$ and $\frac{\partial^2 TC\{p,T\}}{\partial T^2}$.

Step 3.- Here, define Hassian matrix $H = \begin{bmatrix}\frac{\partial^2 TC\{p,T\}}{\partial p^2} & \frac{\partial^2 TC\{p,T\}}{\partial p\partial T}\\ \frac{\partial^2 TC\{p,T\}}{\partial T\partial p} & \frac{\partial^2 TC\{p,T\}}{\partial T^2}\end{bmatrix}$ and find $H_{11} = \frac{\partial^2 TC\{p,T\}}{\partial p^2}$, and $H_{22} = \left(\frac{\partial^2 TC\{p,T\}}{\partial p^2}\right)\left(\frac{\partial^2 TC\{p,T\}}{\partial T^2}\right)-\left(\frac{\partial^2 TC\{p,T\}}{\partial p\partial T}\right)\left(\frac{\partial^2 TC\{p,T\}}{\partial T\partial p}\right)$

Step 4.- Now, check $H_{11} > 0$ and $H_{22} > 0$ then total cost is convex.

Graphical representation-

In this section we represent graphical representation for all different cases with respect to all possible variable.

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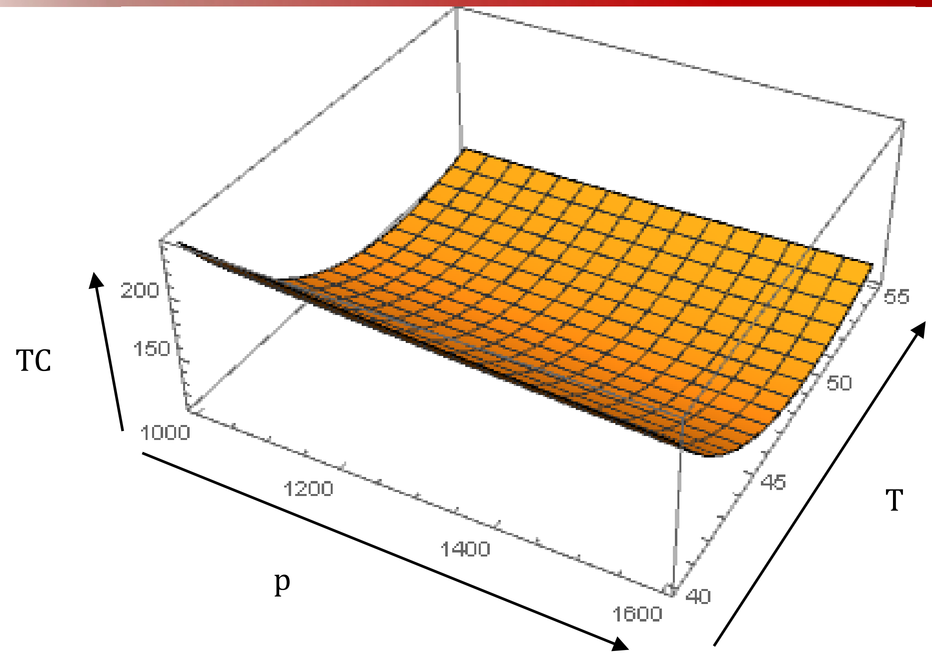

Fig:2- convexity between selling price and total cycle length when $0 \leq M \leq T$.

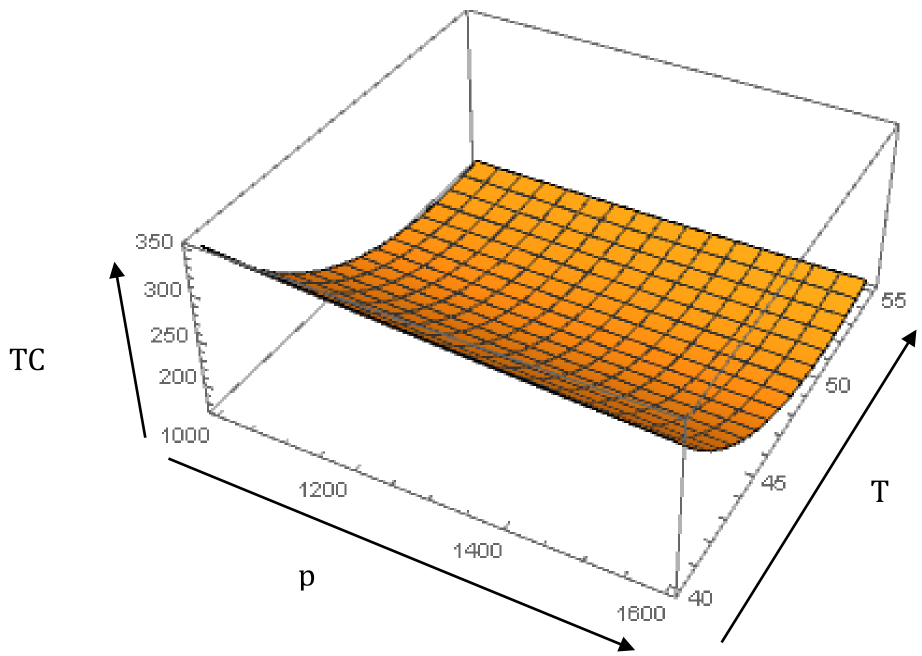

Fig:3- convexity between selling price and total cycle length when $0 \leq T \leq M$.

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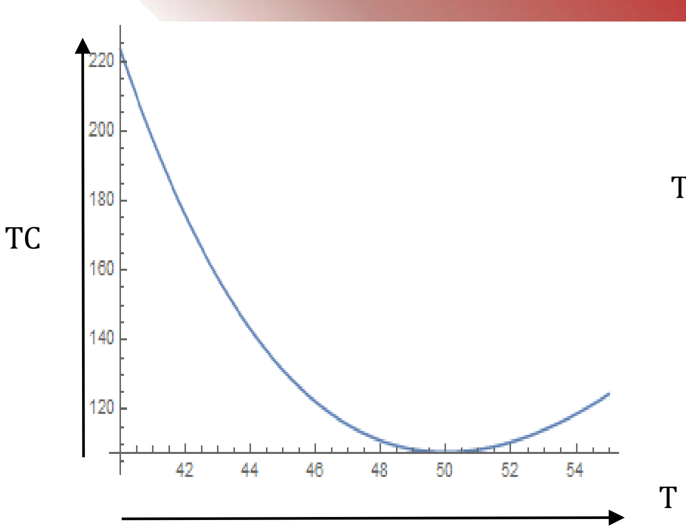


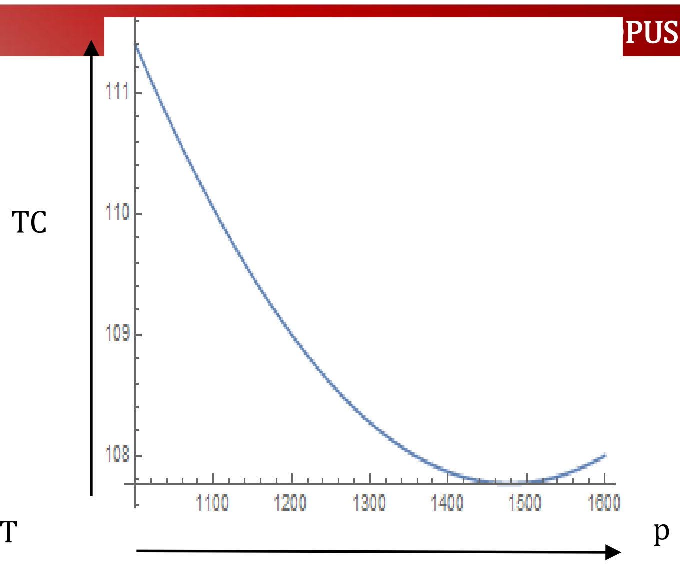
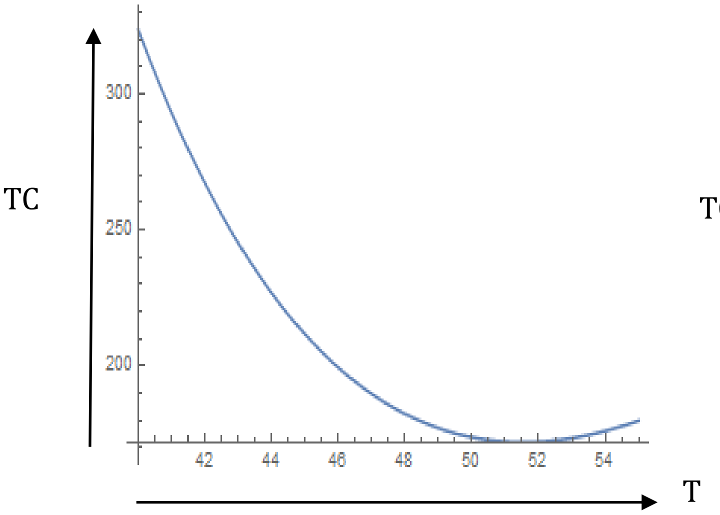


Fig: 4- Graph between total cost and cycle length where x-axis represents cycle length and y-axis represents total cost when $0 \leq M \leq T$.

Fig:5- Graph between total cost and selling price where x-axis represents selling price and y-axis represents total cost when $0 \leq T \leq M$.



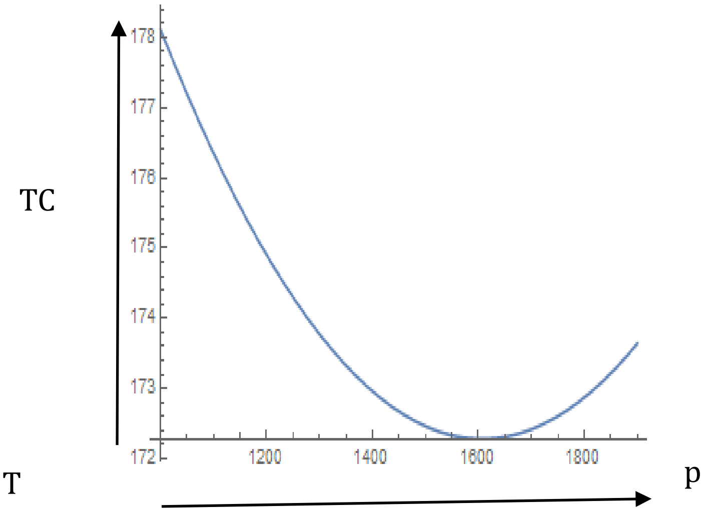


Fig: 6- Graph between total cost and cycle length where x-axis represents cycle length and y-axis represents total cost when $0 \leq M \leq T$.

Fig:7- Graph between total cost and selling price where x-axis represents selling price and y-axis represents total cost when $0 \leq T \leq M$.

Numerical representation-

In this section to illustrate our proposed model the following numerical example is to be considered. Let us assume- y=4, O=\$200/order, C_{h1}= \$ 10/unit/order, ω=0.02, z =1000 unit/days, a=1000, b= 0.1, c= 100, s = \$20 /unit/days, r = 0.01, t_1= 10 days, t_2 = 20 days, t_3 = 35 days, t_4 = 48 days, θ = 0.001 %, C_{h2} = \$20 /unit/order, C_d= \$25 /unit/order, S_c = \$15/unit/order, S_l = \$21 /unit/order, C_0 = \$22 /unit/order, C_p = \$32 /unit/order, C_{e1} = \$5

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/unit/order, C_{e2} = \$30 /unit/order, τ = 0.4 units/days, γ = 0.3 units/days, M = 25 days, ω_1 = 0.45, λ_0 = 0.09, I_e = \$ 0.28 /days, I_c = \$ 0.38 /days. The optimal solution is p = \$1478.62, T = 49.9902 days and total cost = \$ 107.766, which is for case -1 and for case-2 total cost is greater than case-1. So that we done sensitivity analysis for only case-1 with Mathematica 13.0 software.

Sensitivity analysis-

This sensitivity analysis is performed by changing each of the parameters by +50%, +30%, +10%, -10%, -30% and -50%, taking only one parameter at a time and keeping the remaining parameters unchanged. The results are shown in below Table.

Parameters	% change	p	T	Total Cost
a	+20%	1598.9	45.33	109.34
	+10%	1498.4	46.56	108.5
	-10%	1408.3	51.34	106.5
	-20%	1395.5	53.45	105.43
b	+20%	1383.5	57.43	111.3
	+10%	1398.3	54.34	109.4
	-10%	1488.3	46.54	106.54
	-20%	1635.4	44.65	103.4
C_{h1}	+20%	1528.3	49.99	99.54
	+10%	1483.4	49.99	104.5
	-10%	1433.8	49.99	109.45
	-20%	1328.6	49.99	113.44
C_{h2}	+20%	1235.5	48.32	115.4
	+10%	1498.5	48.11	117.4
	-10%	1684.6	49.54	94.34
	-20%	1849.5	50.6	85.44
s	+20%	1452.4	51.34	107.766
	+10%	1478.8	52.34	107.766
	-10%	1487.5	51.23	107.766
	-20%	1499.5	48.34	107.766
r	+20%	1583.5	47.56	101.34
	+10%	1486.5	48.56	104.32
	-10%	1387.6	50.43	108.45
	-20%	1284.5	50.98	109.34
C_d	+20%	1478.62	49.99	110.34
	+10%	1478.62	49.99	108.34
	-10%	1478.62	49.99	105.33
	-20%	1478.62	49.99	102.23
S_c	+20%	1398.5	40.56	98.23
	+10%	1238.3	43.47	93.45
	-10%	1126.4	47.65	105.34
	-20%	1087.5	54.32	112.34
S_l	+20%	1476.34	53.34	116.44
	+10%	1528.4	52.34	113.43
	-10%	1487.3	46.54	104.23
	-20%	1698.4	44.56	99.34

I_e	+20%	1478.62	56.54	89.43
	+10%	1478.62	55.43	95.33
	-10%	1478.62	53.45	116.45
	-20%	1478.62	51.23	121.45
I_c	+20%	1576.4	38.45	107.766
	+10%	1426.5	42.34	107.766
	-10%	1383.5	57.54	107.766
	-20%	1298.5	63.45	107.766
C_p	+20%	1187.4	49.99	118.23
	+10%	1238.4	49.99	114.32
	-10%	1376.3	49.99	105.43
	-20%	1498.5	49.99	101.23
C_{e1}	+20%	1478.2	45.43	107.766
	+10%	1478.3	43.34	107.766
	-10%	1478.7	54.32	107.766
	-20%	1478.9	51.34	107.766
C_{e2}	+20%	1587.4	43.45	98.23
	+10%	1687.4	47.56	93.45
	-10%	1734.5	52.34	105.34
	-20%	1835.3	54.45	112.34
τ	+20%	1478.62	48.5	116.44
	+10%	1478.62	47.6	113.43
	-10%	1478.62	48.65	104.23
	-20%	1478.62	47.65	99.34

Observations-

In this section we

- When increase in parameter a, selling price and total cost is increasing, total cycle length is decreasing.
- When increase in parameter b, selling price decreasing, total cycle length and total cost is increasing.
- When increase in parameter C_{h1}, selling price increasing, total cycle length is constant and total cost fluctuating.
- When increase in parameter C_{h2}, total selling price and total cycle length is decreasing and total cost is decreasing.
- When increase in parameter s, total selling price is decreasing, total cycle length is fluctuating and total cost is constant.
- When increase in parameter r, selling price is increasing, total cycle length and total cost is decreasing.
- When increase in parameter C_d, selling price and total cycle length is constant, total cost is increasing.
- When increase in parameter S_c, selling price is increasing and total cycle length and total coast is decreasing.
- When increase in parameter S_l, selling price is fluctuating, total cycle length and total cost is increasing.

- When increase in parameter I_e, selling price is constant, total cycle length is increasing and total cost is decreasing.
- When increase in parameter I_c, selling price is increasing, total cycle length is decreasing and total cost is constant.
- When increase in parameter C_p, selling price is decreasing, total cycle length is constant and total cost is increasing.
- When increase in parameter C_{e1}, selling price and total cycle length is decreasing and total cost is constant.
- When increase in parameter C_{e2}, selling price, total cycle length and total cost is decreasing.
- When increase in parameter τ, selling price is constant, total cycle length and total cost is increasing.

Conclusion

This study presents a comprehensive inventory model that integrates the challenges of deteriorating items, variable demand influenced by time and price, trade-credit policies, and the impact of inflation. Additionally, it incorporates an inspection policy to account for imperfections in inventory, ensuring higher product quality and minimizing losses due to defective items. The proposed model provides actionable insights into optimizing inventory levels, balancing inspection costs, and managing financial constraints under trade-credit terms. Through its holistic approach, the model bridges several gaps in existing research, offering a practical framework for businesses dealing with complex, real-world inventory challenges.

The results underscore the importance of considering all interdependent factors deterioration, demand variability, trade-credit, inflation, and inspection when making inventory decisions. The findings suggest that businesses can significantly reduce costs and improve operational efficiency by adopting tailored inventory policies that account for these dynamics.

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