Corrosion Management
(https://corrosion-management.com/)
Volume 34, Issue 02, July– Dec 2024
ISSN: 1355-5243
SCOPUS

MANPOWER ANALYSIS USING NON-STATIONARY POISSON PROCESS UNDER GRADED SYSTEM

K. Srinivasa Rao [1], K. Suryanarayana Rao [2 *]

[1] *Department of Statistics, Andhra University, Visakhapatnam, India. Email: ksraoau@yahoo.co.in*
[2] *Department of Basic sciences and Humanities, Vignan's Institute of Engineering for Women, Visakhapatnam, India. Email: suryanarayanarao1@gmail.com*
[*] *Corresponding Author: suryanarayanarao1@gmail.com*

Abstract: Manpower analysis is most important for any organization to utilize the resources more effectively. In this article we developed and analyzed a stochastic model for two graded system with the consideration that the recruitment, promotion and leaving processes follow Non-Homogeneous Poisson processes. The Non-Homogeneous Poisson processes closely matches with the statistical characteristics are the recruitment, promotion and leaving processes for organization such as corporates and private organizations where these processes are sensitive towards time using the joint p.g.f the model characteristics are derived and analyzed. It is observed that the measures of the HR system in the organization such as expected number of employees in each grade, the variability of the grade size distribution, the C.V in each grade are sensitive to the parameters of the recruitment, promotion and leaving. This model also includes the stationary model as a particular case for specific values of the parameter. This model is much useful for HR analytics in predicting the human resource management performance and developing the policies for optimal utilization of manpower.

Keywords: Human resource management, non-stationary process, intermediary leavings, performance evaluation, Non-Homogeneous Poisson process, sensitivity analysis.

1. INTRODUCTION

Human resource management is pre-requisite for sustainable development of any organization. A critical component of present-day human resources management is manpower planning. The objective of manpower planning is to create strategies that address future demand for human resources. Of late a lot of focus has been placed on modelling the manpower system in resource allocation and establishing strategies for human resource development. The concept of labor turnover was investigated using demography analogy by Silcock [1]. Bartholomew [2-3] studied manpower models using the idea that an employee's entire duration of service in a company is unpredictable and follows a probability distribution. Ugwuowo and Mc Clean [4]; Wang [5] examined the manpower models with the help of the various techniques used for model construction and analysis. Kannan Nilkantan [6] analyzed the manpower models with staffing policies. Manpower models governed by a fuzzy environment were studied (Jeeva and Geetha [7]; Gulzarul Hasan and Suhaib Hasan[8]; Gulzarul Hasan et al.[9]). Osagide and Ekhosuehi [10] continuously examined manpower models using sparse stochastic matrices, creating a Markov chain. Tames Banyai et al. [11] studied the markov chain approach for human resources deployment. Bilkisu Maijamma et al. [12] studied linear programming methods for recruitment and promotion policies. Vincent et al. [13] examined the literature on personnel planning with a focus on manpower systems developed in the setting of Markov chains.

Anantharaj [14] studied the expected time to recruitment using shock model under the assumption that the wastages in subsequent decision epochs are correlated random variables. Saral et al. [15] studied manpower models with two graded systems with respect to recruitment policy and thresholds. Sivasamy et al. [16] studied manpower of an organization where the "exits" of engaged workers result in some loss or wastage. Ravichandran and Srividhya [17] considered a labour structure for one grade in which employees leave in groups at random epochs. Using the CUM and MAX recruiting techniques, two stochastic examples are built. Mochammad et al. [18] investigated how job satisfaction influences employees intentions to leave the firm, which are influenced by organizational commitment. Srinivasa Rao and Ganapathi Swamy [19]; Ganapathi Swamy and Srinivasa Rao [20] have studied manpower models with Duane recruitment processes. In these papers they assumed that the recruitment is time dependent and non-stationary. But no serious attempt is made to study manpower models with non stationary recruitment, leaving and promotion processes which are quit common in many organizations due to time dependent nature of the organizations policies. In human resource models with the intermediary exits and non-homogeneous hiring, promoting and leaving procedures in graded systems, very little research has been documented. Hence in this study we design and analyze a model with intermediate departures and NHP processes for hiring, promoting and leaving processes. This model is useful for designing strategies at corporate offices and other private sector organizations.

Arrangement of the remaining paper as follows: Section 2 uses difference differential equations to build the two graded manpower model. Section 3 focuses on the determination of model characteristics, such as extinction probability, in grade 1 and grade 2, the probability of at least one employee in each grade, average number of employees in each grade. Section 4 focuses on numerical demonstration and discussion of the model characteristics. Section 5 deals with sensitivity analysis of the model. Section 6 compares the suggested model with that of other models. Section 7 addresses the conclusions.

Corrosion Management ISSN: 1355-5243
(https://corrosion-management.com/)
Volume 34, Issue 02, July– Dec 2024
SCOPUS

2. TWO GRADED MANPOWER MODEL

In this section, a manpower model is taken into account where an organization has two grades. A two-graded model is considered where intermediatory departures are permitted. Employees are always hired in grade-1 at every recruitment. In grade 1, it is anticipated that the recruitment process will follow a NHP process with a mean recruitment rate of $\lambda(t) = \lambda_1 + \lambda_2 t$. The promotion procedure from grade-1 to grade-2 with a mean promotion rate $\alpha(t) = a_1 + a_2 t$. The procedures of leaving in grade 2 with a mean leaving rate of $\beta(t) = b_1 + b_2 t$. Additionally, it is assumed that an employee after spending have some random amount of time in grade 1 he/she may leave the organization or join the 2^{nd} grade . The probability of an employee leaving the organization after the first grade is π and promoting to the grade-2 is $\theta = 1\text{-}\pi$. Furthermore, it is assumed that the promotion and leaving processes one NHP processes.

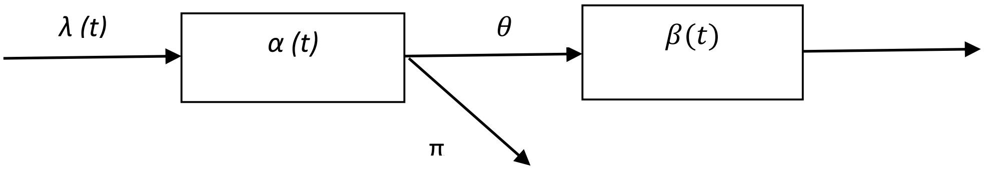

Figure 1. Manpower Model

With these considerations, the model postulates are:

- Events occurring at distinct times are statistically independent.
- The probability of an employee being placed in grade-1 at an interval of time 'h' is [$\lambda\ (t)\ h + o(h)$].
- The chance of a promotion from grade 1 to grade 2 during a period of time 'h' when there are 'n' employees in grade 1 is [$n\ \alpha(t)\ h + o(h)$].
- When there are 'm' employees in grade 2, the probability of an employee quitting the company from grade-2 during an interval of time 'h' is [$m\ \beta(t)\ h + o(h)$].
- The chances of a grade 1 employee quitting the company with a specific probability within an infinitesimally small period of time "h" is [$n\ \alpha(t)\ \pi\ h + o(h)$].
- The chance that an employee will be promoted from grade 1 to grade 2 over an infinitesimally small period of time 'h' is [$n\ \alpha(t)\ \theta\ h + o(h)$], where $\theta + \pi = 1$.
- The chance that no employees will join or quit the company during the period of time 'h' when there are 'n' employees in grade 1 and 'm' employees in grade 2 is [$1 - \lambda(t)h - n\ \mu(t)h - m\ \beta(t)\ h + o(h)$].
- The chance that an event occurs during a small period of time "h" is $o(h)$.

Let $P_{n,m}$ (t) represent the probability that the organization will have 'n' employees in grade-1 and 'm' employees in grade-2 at time t. The difference-differential equations of the system are:

$$\frac{\partial P_{n,m}(t)}{\partial t} = -[\lambda(t) + n\,\alpha(t) + m\beta(t)]p_{n,m}(t) + \lambda(t)P_{n-1,m}(t) + (n+1)\alpha(t)\theta P_{n+1,m-1}(t)$$
$$+(n+1)\alpha(t)\pi P_{n+1,m}(t) + (m+1)\,\beta(t)\,P_{n,m+1}(t)\,;\ \ \forall\, n,m \geq 0 \qquad (1)$$

$$\frac{\partial P_{n,0}(t)}{\partial t} = -\,[\lambda(t) + n\,\alpha(t)]\,P_{n,0}(t) + \lambda(t)\,P_{n-1,0}(t) + (n+1)\alpha(t)\pi P_{n+1,0}(t)$$
$$+\beta(t)P_{n,1}(t)\,;\ \ \forall\, n > 0, m = 0 \qquad (2)$$

$$\frac{\partial P_{0,m}(t)}{\partial t} = -\,[\lambda(t) + m\,\beta(t)]P_{0,m}(t) + \alpha(t)\theta P_{1,m-1}(t) + \alpha(t)\pi P_{1,m}(t)$$
$$+\,(m+1)\beta(t)P_{0,m+1}(t)\,;\ \ \forall\, n = 0, m > 0 \qquad (3)$$

$$\frac{\partial P_{0,0}(t)}{\partial t} = -[\lambda(t)]P_{0,0}(t) + \alpha(t)\pi P_{1,0}(t) + \beta(t)\,P_{0,1}(t);\ \ \forall\, n = 0, m = 0 \qquad (4)$$

Let $P(Z_1, Z_2; t)$ be the joint p.g.f of $P_{n,m}(t)$. Then

$$P(Z_1, Z_2; t) = \sum_{n=0}^{\infty}\sum_{m=0}^{\infty} p_{n,m}(t){z_1}^{n}{z_2}^{m} \qquad (5)$$

This entails

$$\frac{\partial P_{n,m}(t)}{\partial t} = -\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}[\lambda(t) + n\,\alpha(t) + m\,\beta(t)]p_{n,m}(t){z_1}^{n}{z_2}^{m}$$
$$+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}\lambda(t)P_{n-1,m}(t){z_1}^{n}{z_2}^{m} + \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(n+1)\alpha(t)\theta P_{n+1,m-1}(t){z_1}^{n}{z_2}^{m}$$
$$+\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(n+1)\alpha(t)\pi\, P_{n+1,m}(t){z_1}^{n}{z_2}^{m} + \sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(m+1)\beta(t)P_{n,m+1}(t)z_1^{n}z_2^{m} \qquad (6)$$

This entails

$$\frac{\partial P(Z_1, Z_2; t)}{\partial t} = [\alpha(t)(1-\theta-z_1+\theta z_2)]\frac{\partial P}{\partial z_1} + [\beta(t)(1-z_2)]\frac{\partial P}{\partial z_2} - \lambda(t)(1-z_1)P \qquad (7)$$

The auxiliary equations are:

$$\frac{dt}{1} = \frac{dz_1}{-\alpha(t)(1-\theta-z_1+\theta\, z_2)} = \frac{dz_2}{-\beta(t)\,(1-z_2)} = \frac{dP}{-\lambda(t)(1-z_1)P(z_1, z_2, t)} \qquad (8)$$

Consider the recruitment, promotion and departure rates are:

$\lambda(t) = \lambda_1 + \lambda_2 t$

$\alpha(t) = a_1 + a_2 t$, Where $a_1 > 0$, $a_2 > 0$

$\beta(t) = b_1 + b_2 t$, Where $b_1 > 0$, $b_2 > 0$

First and third terms in equation (8), will give

$$A = (z_2 - 1)e^{-\int \beta(t)dt} \qquad (9)$$

The first and second terms in equation (8), yields

$$B = z_1 e^{-\int \alpha(t)dt} + (z_2 - 1)e^{-\int \beta(t)dt}\theta\left(\int \alpha(t)e^{\int[\beta(t)-\alpha(t)]dt}\,dt\right) + \int \alpha(t)e^{-\int \alpha(t)dt}\,dt \qquad (10)$$

The first and fourth terms in equation (8), yields

$$C = P(z_1, z_2; t)\exp(-[z_1\, e^{-\int \alpha(t)dt} + (z_2 - 1)\, e^{-\int \beta(t)dt}\theta \left(\int \alpha(t)\, e^{\int[\beta(t)-\alpha(t)]dt}\, dt\right)$$

$$+ \int \alpha(t).\, e^{-\int \alpha(t)dt}\, dt]\, [\int \lambda(t).\, e^{\int \alpha(t)dt}\, dt])$$

$$+ [(z_2 - 1)\, e^{-\int \beta(t)dt}\theta \int \lambda(t).\, e^{\int \alpha(t)dt} (\int \alpha(t)\, e^{\int[\beta(t)-\alpha(t)]dt}\, dt)dt]$$

$$+ [\int \lambda(t).\, e^{\int \alpha(t)dt} (\int \alpha(t)\, e^{-\int \alpha(t)dt}\, dt)dt] + \int \lambda(t)\, dt \tag{11}$$

Where arbitrary constants A, B and C are used. With the initial conditions $P_{00}(0)=1$, $P_{00}(t) = 0$, $\forall\, t > 0$.

The joint p.g.f. of the number of employees in grades 1 and 2 at time 't' is

$$P(z_1, z_2; t) = exp[\lambda_1[(z_1 - 1)e^{-\left(a_1 t + a_2\frac{t^2}{2}\right)\left(\frac{\int_0^t (\lambda_1+\lambda_2 v)e^{\left(a_1 v + a_2\frac{v^2}{2}\right)}dv}{\lambda_1} - \frac{1}{a_1}\right)}$$

$$+\theta\,(z_2 - 1)e^{-\left(b_1 t + b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1 - a_1} - \frac{\int_0^t (a_1 + a_2 v)\, e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}\, dv}{a_1}\right)$$

$$+\theta(z_2 - 1)e^{-\left(b_1 t + b_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t (\lambda_1+\lambda_2 v)e^{\left(a_1 v + a_2\frac{v^2}{2}\right)}dv \int_0^t (a_1 + a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}\, dv}{\lambda_1}\right.$$

$$\left. - \frac{\int_0^t (\lambda_1+\lambda_2 v)e^{\left(a_1 v + a_2\frac{v^2}{2}\right)}dv \left(\int_0^t (a_1 + a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}\, dv\right)dv}{\lambda_1} - \frac{1}{b_1}\right)]] \tag{12}$$

3. CHARACTERISTICS OF THE MODEL

The probability that there are no employees in the organization is

$$P_{0,0}(t) = exp[-\lambda_1[e^{-\left(a_1 t + a_2\frac{t^2}{2}\right)\left(\frac{\int_0^t (\lambda_1+\lambda_2 v)e^{\left(a_1 v + a_2\frac{v^2}{2}\right)}dv}{\lambda_1} - \frac{1}{a_1}\right)}$$

$$+\ \theta\, e^{-\left(b_1 t + b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1 - a_1} - \frac{\int_0^t (a_1 + a_2 v)\, e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}\, dv}{a_1}\right)$$

$$+\theta e^{-\left(b_1 t + b_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t (\lambda_1+\lambda_2 v)e^{\left(a_1 v + a_2\frac{v^2}{2}\right)}dv \int_0^t (a_1 + a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}\, dv}{\lambda_1}\right.$$

 ISSN: 1355-5243

$$-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}\left(\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}dv}\right)dv}{\lambda_1}-\frac{1}{b_1}\Big)\Big]\Big] \quad (13)$$

Taking $z_2 = 1$ in $P(z_1, z_2; t)$, the p.g.f. for the number of employees in grade-1 is

$$P(z_1,t) = exp\left[\lambda_1(z_1-1)e^{-\left(a_1 t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)\ e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)\right] \quad (14)$$

The probability that there is no employee in grade -1 of the organization is

$$P_{0.}(t) = exp\left[-\lambda_1 e^{-\left(a_1 t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)\ e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)\right] \quad (15)$$

The organization's average number of employees in grade-1 is

$$L_1(t) = \lambda_1 e^{-\left(a_1 t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)\ e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right) \quad (16)$$

The probability that the organization has at least one grade-1 employee is

$$U_1(t) = 1 - P_{0.}(t)$$

$$= 1 - \exp\left[-\lambda_1 e^{-\left(a_1 t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)\ e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)\right] \quad (17)$$

The average wait time for an employee in the organization's grade-1 level is

$$W_1(t) = \frac{L_1(t)}{\alpha(t)[1-P_{0.}(t)]}$$

$$= \frac{\lambda_1 e^{-\left(a_1 t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)\ e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)}{(a_1+a_2 t)\left[1-exp\left[-\lambda_1 e^{-\left(a_1 t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)\ e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)\right]\right]} \quad (18)$$

In grade-1the variance of the number of employees

$$V_1(t) = \lambda_1 e^{-\left(a_1 t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)\ e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right) \quad (19)$$

In grade-1 the C.V of number of employees is

$$CV_1(t) = \frac{\sqrt{V_1(t)}}{L_1(t)}$$

ISSN: 1355-5243

$$= \left[\lambda_1 e^{-\left(a_1 t + a_2 \frac{t^2}{2}\right)}\left(\frac{\int_0^t (\lambda_1+\lambda_2 v)\ e^{\left(a_1 v + a_2 \frac{v^2}{2}\right)} dv}{\lambda_1} - \frac{1}{a_1}\right)\right]^{\frac{-1}{2}} \tag{20}$$

The p.g.f of the number of employees in grade-2 is

$$P(z_2,t) = exp[\lambda_1\theta\, [(z_2-1)e^{-\left(b_1 t + b_2 \frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1} - \frac{\int_0^t (a_1+a_2 v).\ e^{(b_1-a_1)v + (b_2-a_2)\frac{v^2}{2}} dv}{a_1}\right)$$

$$+(z_2-1)e^{-\left(b_1 t + b_2 \frac{t^2}{2}\right)}\left(\frac{\int_0^t (\lambda_1+\lambda_2 v)e^{\left(a_1 v + a_2 \frac{v^2}{2}\right)} dv \int_0^t (a_1+a_2 v)e^{(b_1-a_1)v + (b_2-a_2)\frac{v^2}{2}} dv}{\lambda_1}\right.$$

$$\left. - \frac{\int_0^t (\lambda_1+\lambda_2 v)e^{\left(a_1 v + a_2 \frac{v^2}{2}\right)}\left(\int_0^t (a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}} dv\right) dv}{\lambda_1} - \frac{1}{b_1}\right)]] \tag{21}$$

The probability that there is no employee in grade -2 is

$$P_{.0}(t) = exp[-\lambda_1\theta[e^{-\left(b_1 t + b_2 \frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1} - \frac{\int_0^t (a_1+a_2 v).\ e^{(b_1-a_1)v + (b_2-a_2)\frac{v^2}{2}} dv}{a_1}\right)$$

$$+e^{-\left(b_1 t + b_2 \frac{t^2}{2}\right)}\left(\frac{\int_0^t (\lambda_1+\lambda_2 v)e^{\left(a_1 v + a_2 \frac{v^2}{2}\right)} dv \int_0^t (a_1+a_2 v)e^{(b_1-a_1)v + (b_2-a_2)\frac{v^2}{2}} dv}{\lambda_1}\right.$$

$$\left. - \frac{\int_0^t (\lambda_1+\lambda_2 v)e^{\left(a_1 v + a_2 \frac{v^2}{2}\right)}\left(\int_0^t (a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}} dv\right) dv}{\lambda_1} - \frac{1}{b_1}\right)]] \tag{22}$$

The organization's average number of grade-2 employees is

$$L_2(t) = \lambda_1\theta\ e^{-\left(b_1 t + b_2 \frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1} - \frac{\int_0^t (a_1+a_2 v).\ e^{(b_1-a_1)v + (b_2-a_2)\frac{v^2}{2}} dv}{a_1}\right)$$

$$+\lambda_1\theta e^{-\left(b_1 t + b_2 \frac{t^2}{2}\right)}\left[\frac{\int_0^t (\lambda_1+\lambda_2 v)e^{\left(a_1 v + a_2 \frac{v^2}{2}\right)} dv \int_0^t (a_1+a_2 v)e^{(b_1-a_1)v + (b_2-a_2)\frac{v^2}{2}} dv}{\lambda_1}\right.$$

$$\left. - \frac{\int_0^t (\lambda_1+\lambda_2 v)e^{\left(a_1 v + a_2 \frac{v^2}{2}\right)}\left(\int_0^t (a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}} dv\right) dv}{\lambda_1} - \frac{1}{b_1}\right] \tag{23}$$

The probability that the organization has at least one employee in grade-2 is

$U_2(t) = 1 - P_{.0}(t)$

$$=1 - exp[-\lambda_1\theta[e^{-\left(b_1 t + b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1} - \frac{\int_0^t (a_1+a_2v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)$$

$$+ e^{-\left(b_1 t + b_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}\left(\int_0^t(a_1+a_2v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1} - \frac{1}{b_1}\right)\Big]\Big] \quad (24)$$

The average wait time for an employee in grade-2 is

$$W_2(t) = \frac{L_2(t)}{(b_1+b_2t)[\,U_2(t)]}$$

$L_2(t)$ and $U_2(t\,)$ are given in equations (23) and (24).

In grade-2 the variance of the number of employees is

$$V_2(t) = \lambda_1\theta\, e^{-\left(b_1 t + b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1} - \frac{\int_0^t (a_1+a_2v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)$$

$$+\lambda_1\theta\, e^{-\left(b_1 t + b_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}dv\left(\int_0^t(a_1+a_2v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1} - \frac{1}{b_1}\right] \quad (25)$$

In grade-2 the C.V of number of employees is

$$CV_2(t) = \left[\lambda_1\theta e^{-\left(b_1 t + b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1} - \frac{\int_0^t (a_1+a_2v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)\right.$$

$$+\lambda_1\theta e^{-\left(b_1 t + b_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}\right.$$

Corrosion Management ISSN: 1355-5243
(https://corrosion-management.com/)
Volume 34, Issue 02, July– Dec 2024
SCOPUS

$$-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\left(\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\Bigg]\Bigg]^{\frac{-1}{2}} \tag{26}$$

The mean number of employees in the organization is

$$L(t) = L_1(t) + L_2(t)$$

$$= \lambda_1 e^{-\left(a_1 t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)\, e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)$$

$$+\ \lambda_1\theta\, e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1}-\frac{\int_0^t(a_1+a_2 v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)$$

$$+\ \lambda_1\theta e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\Bigg[\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}$$

$$-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\left(\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\Bigg] \tag{27}$$

The variance of the number of employees in the organization is

$$V(t) = V_1(t) + V_2(t)$$

$$= \lambda_1 e^{-\left(a_1 t+a_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2 v)\, e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv}{\lambda_1}-\frac{1}{a_1}\right)$$

$$+\ \lambda_1\theta\, e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\left(\frac{1}{b_1-a_1}-\frac{\int_0^t(a_1+a_2 v).\ e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{a_1}\right)$$

$$+\ \lambda_1\theta\, e^{-\left(b_1 t+b_2\frac{t^2}{2}\right)}\Bigg[\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv}{\lambda_1}$$

$$-\frac{\int_0^t(\lambda_1+\lambda_2 v)e^{\left(a_1 v+a_2\frac{v^2}{2}\right)}dv\left(\int_0^t(a_1+a_2 v)e^{(b_1-a_1)v+(b_2-a_2)\frac{v^2}{2}}dv\right)dv}{\lambda_1}-\frac{1}{b_1}\Bigg] \tag{28}$$

ISSN: 1355-5243

4. NUMERICAL ILLUSTRATION OF THE MODEL

The model performance characteristics are extremely time sensitive, the behavior of the model is investigated by computing measures of performance using the following values for the model parameters: t = 0.13, 0.14, 0.15, 0.16; λ_1 = 2, 3, 4, 5, 6; λ_2 = 3, 4, 5, 6, 7; a_1 = 7, 7.4, 7.8, 8.2, 8.6; a_2 = 5, 7, 9, 11, 13; b_1 = 9, 9.4, 9.8, 10.2, 10.6; b_2 = 9, 12, 15, 18, 21 ; θ = 0.1,0.2,0.3,0.4,0.5; and π = 0.9,0.8,0.7,0.6,0.5.

The performance measures, such the organization's average number of employees in grades 1 and 2, the average wait time for an employee in grades 1 and 2, the variance of number of employees in grades 1 and 2, the C.V of the number of employees in grades 1 and 2 are computed and shown in Table 1, Table 2, Figure 1, Figures 2a, 2b and Figures 3a, 3b.

From Table 1, as time (t) varies from 0.13 to 0.16, the average number of employees in grade-1 increases from 0.07505 to 0.12475 and in grade 2 it decreases from 0.01331 to 0.07818, the average waiting time of an employee in grade-1 increases from 0.13569 to 0.13637 and in grade-2 it increases from 0.09898 to 0.09616, when other parameters are fixed.

When other parameter are held constant and the recruitment rate (λ_1) varies from 3 to 6, the average number of grades 1 and 2 employees increase from 0.17384 to 0.32109 and 0.01173 to 0.02345 respectively, and the average waiting time of an employee in each grade increase from 0.13967 to 0.14989 and 0.09635 to 0.09691 respectively.

When all other parameters are held constant and the recruitment rate (λ_2) varies from 4 to 7, the average number of grade 1 employees increases from 0.32995 to 0.35654 while it stays the same in grade 2, the average waiting time of an employee in grade 1increases from 0.15052 to 0.15242 while it remains same in grade 2.

When the rest of the parameters are held constant and the promotion rate parameter (a_1) changes from 7.4 to 8.6, the average number of grades 1 and 2 employees increase from 0.37094 to 0.39615 and from 0.04024 to 0.28033 respectively. The average waiting time for an employee in grades 1 and 2 also changes from 0.14596 to 0.12884 and from 0.09773 to 0.10984 respectively.

The average number of grades 1 and 2 employees decrease from 0.39277 to 0.38282 and from 0.28013 to 0.27958 respectively as the promotion rate parameter (a_2) varies from 7 to 13, and all other parameters remain constant, the average waiting time of an employee in each grade decrease from 0.12440 to 0.11270 and from 0.10983 to 0.10980.When the other parameters are fixed and the average number of grade-1 employees stays constant while that in grade-2 decreases from 0.11353 to 0.01933. As the leaving rate parameter (b_1) changes from 9.4 to 10.6, and that the average waiting time for an employee in grade-1 stays the same while that of grade-2 decreases from 0.09804 to 0.09159.

Corrosion Management
(https://corrosion-management.com/)
Volume 34, Issue 02, July– Dec 2024

ISSN: 1355-5243

SCOPUS

Table 1. Value of $L_1(t)$, $L_2(t)$, $W_1(t)$ and $W_2(t)$ for various value of parameters.

t	λ_1	λ_2	a_1	a_2	b_1	b_2	Θ	π	$L_1(t)$	$L_2(t)$	$W_1(t)$	$W_2(t)$
0.13	2	3	7	5	9	9	0.1	0.9	0.07505	0.01331	0.13569	0.09898
0.14	2	3	7	5	9	9	0.1	0.9	0.09266	0.01124	0.13598	0.09801
0.15	2	3	7	5	9	9	0.1	0.9	0.10921	0.00941	0.13621	0.09707
0.16	2	3	7	5	9	9	0.1	0.9	0.12475	0.00781	0.13637	0.09616
0.16	**3**	3	7	5	9	9	0.1	0.9	0.17384	0.01173	0.13967	0.09635
0.16	**4**	3	7	5	9	9	0.1	0.9	0.22292	0.01564	0.14303	0.09654
0.16	**5**	3	7	5	9	9	0.1	0.9	0.27200	0.01955	0.14643	0.09672
0.16	**6**	3	7	5	9	9	0.1	0.9	0.32109	0.02345	0.14989	0.09691
0.16	6	**4**	7	5	9	9	0.1	0.9	0.32995	0.02345	0.15052	0.09691
0.16	6	**5**	7	5	9	9	0.1	0.9	0.33881	0.02345	0.15115	0.09691
0.16	6	**6**	7	5	9	9	0.1	0.9	0.34767	0.02345	0.15178	0.09691
0.16	6	**7**	7	5	9	9	0.1	0.9	0.35654	0.02345	0.15242	0.09691
0.16	6	7	**7.4**	5	9	9	0.1	0.9	0.37094	0.04024	0.14596	0.09773
0.16	6	7	**7.8**	5	9	9	0.1	0.9	0.38196	0.06753	0.13990	0.09906
0.16	6	7	**8.2**	5	9	9	0.1	0.9	0.39020	0.12117	0.13419	0.10171
0.16	6	7	**8.6**	5	9	9	0.1	0.9	0.39615	0.28033	0.12884	0.10984
0.16	6	7	8.6	**7**	9	9	0.1	0.9	0.39277	0.28013	0.12440	0.10983
0.16	6	7	8.6	**9**	9	9	0.1	0.9	0.38942	0.27994	0.12025	0.10982
0.16	6	7	8.6	**11**	9	9	0.1	0.9	0.38611	0.27975	0.11636	0.10981
0.16	6	7	8.6	**13**	9	9	0.1	0.9	0.38282	0.27958	0.11270	0.10980
0.16	6	7	8.6	13	**9.4**	9	0.1	0.9	0.38282	0.11353	0.11270	0.09804
0.16	6	7	8.6	13	**9.8**	9	0.1	0.9	0.38282	0.05981	0.11270	0.09318
0.16	6	7	8.6	13	**10.2**	9	0.1	0.9	0.38282	0.03403	0.11270	0.09098
0.16	6	7	8.6	13	**10.6**	9	0.1	0.9	0.38282	0.01933	0.11270	0.09159
0.16	6	7	8.6	13	10.6	**12**	0.1	0.9	0.38282	0.01832	0.11270	0.07632
0.16	6	7	8.6	13	10.6	**15**	0.1	0.9	0.38282	0.01735	0.11270	0.06786
0.16	6	7	8.6	13	10.6	**18**	0.1	0.9	0.38282	0.01642	0.11270	0.06210
0.16	6	7	8.6	13	10.6	**21**	0.1	0.9	0.38282	0.01552	0.11270	0.05772
0.16	6	7	8.6	13	10.6	21	**0.2**	**0.8**	0.38282	0.03105	0.11270	0.05828
0.16	6	7	8.6	13	10.6	21	**0.3**	**0.7**	0.38282	0.04657	0.11270	0.05885
0.16	6	7	8.6	13	10.6	21	**0.4**	**0.6**	0.38282	0.06210	0.11270	0.05942
0.16	6	7	8.6	13	10.6	21	**0.5**	**0.5**	0.38282	0.07762	0.11270	0.05999

Corrosion Management
(https://corrosion-management.com/)
Volume 34, Issue 02, July– Dec 2024
ISSN: 1355-5243
SCOPUS

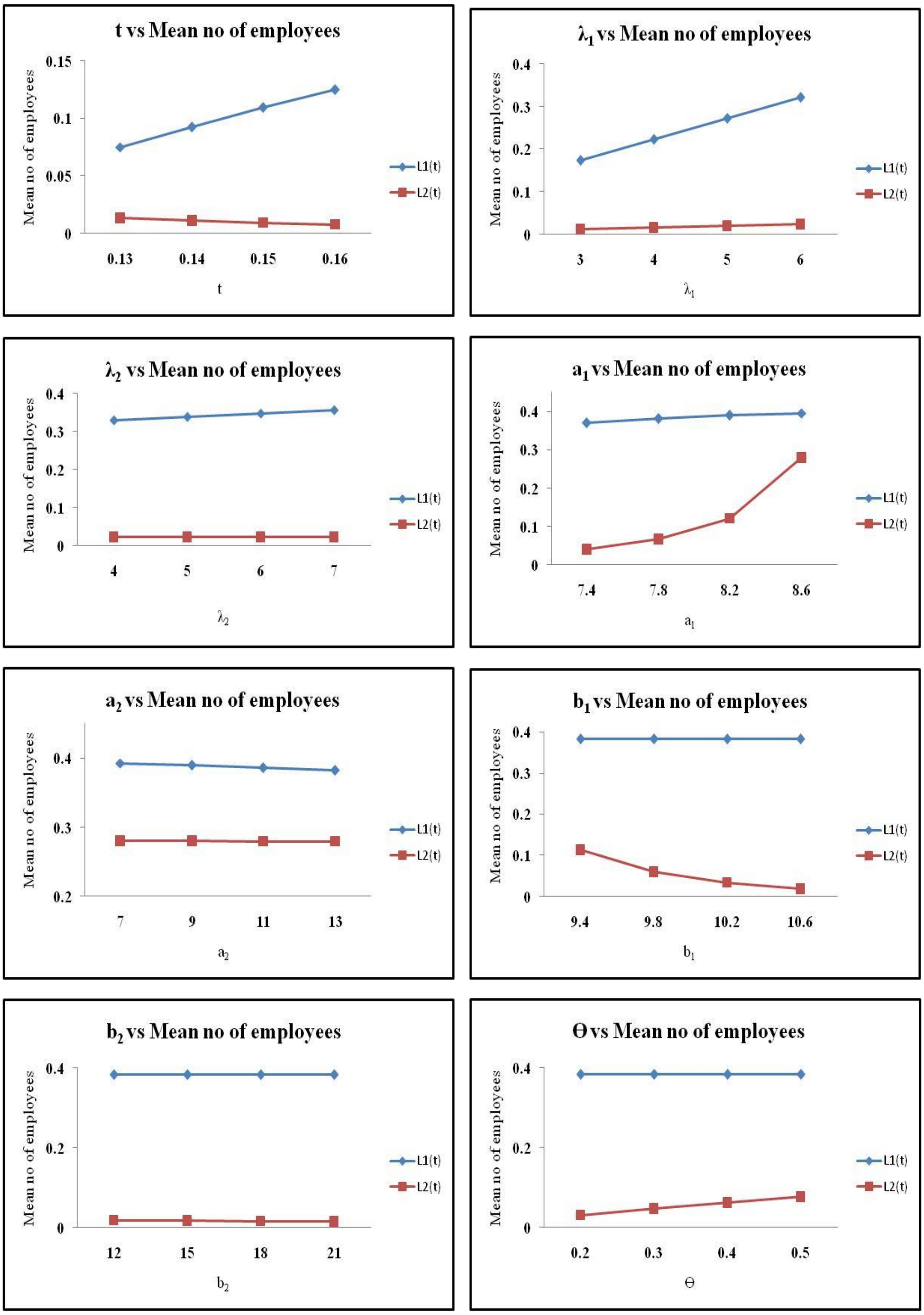

Figure 2a. Relationship between the performance metrics and the parameters.

Corrosion Management
(https://corrosion-management.com/)
Volume 34, Issue 02, July– Dec 2024
ISSN: 1355-5243
SCOPUS

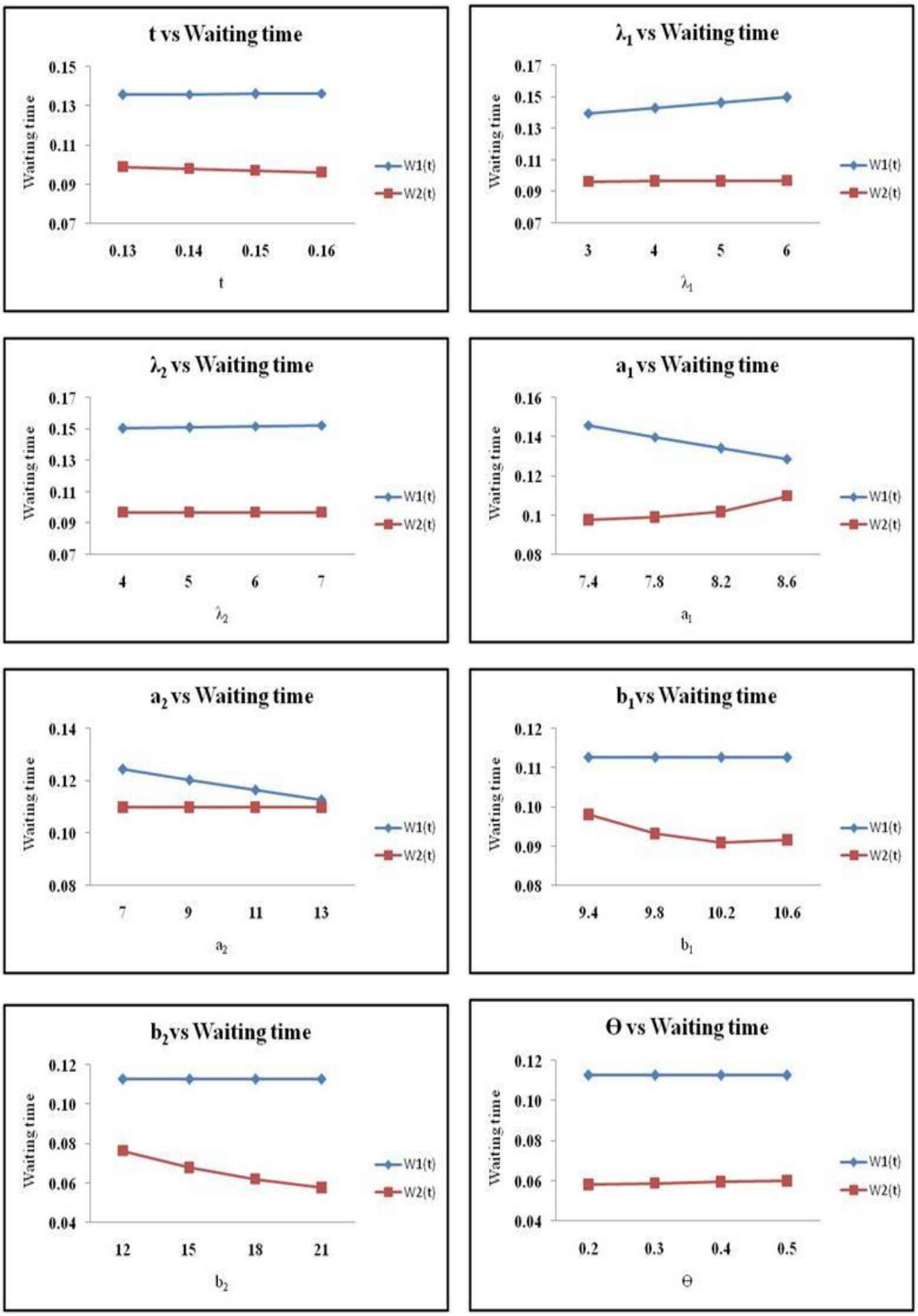

Figure 2b. Relationship between the performance metrics and the parameters

When all other parameters are held constant and the promotion rate parameter (b_2) varies from 12 to 21, the average number of grade-1 employees remains constant and in grade-2 it decreases from 0.01832 to 0.01552, and the average waiting time of an employee in grade 1 does not change and in grade-2 decreases from 0.07632 to 0.05772.

When all other parameters are held constant and the promotion rate parameter (θ) varies from 0.2 to 0.5, the average number of grade-1 employees remains constant and in grade-2 it increases from 0.03105 to 0.07762, while the average waiting time of an employee in grade-1 remains constant and in grade-2 it increases from 0.05885 to 0.05999.

From Table 2, when time (t) varies over 0.13 to 0.16, in grade-1 the variance of number of employees increases from 0.07505 to 0.12475 and in grade-2 falls from 0.01331 to 0.00781, in grade-1 the C.V of number of employees decreases from 3.65017 to 2.83122 and in grade-2 it increases from 8.66901 to 11.3096.

When all other parameters are held constant, the recruitment rate parameter (λ_1) changes from 3 to 6, the variance of the number of employees in grades 1 and 2 are increasing from 0.17384 to 0.32109 and 0.01173 to 0.02345 respectively. The C.V of the number of employees in grades 1 and 2 decrease from 2.39844 to 1.76477 and 9.23427 to 6.52962 respectively.

When all other parameters are held constant, the recruitment rate parameter (λ_2) varies from 4 to 7, the variance of the number of employees in grade 1 is increasing from 0.32995 to 0.35654 and in grade 2 it remains constant. The C.V of the number of employees in grade 1 it decreases from 1.74091 to 1.67474 and in grade 2 stay constant.

As the promotion rate parameter (a_1) changes from 7.4 to 8.6, the variance of the number of employees increase in grades 1 and 2 increases from 0.37094 to 0.39615 and 0.04024 to 0.28033, respectively. The C.V of number of employees in grades 1 and 2 decrease from 1.64191 to 1.58881 and 4.98511 to 1.88870 respectively.

When the rest of the parameters are held constant and the promotion rate parameter (a_2) varies from 7 to 13, the variance of the number of employees in grades 1 and 2 reduce from 0.39277 to 0.38282 and 0.28013 to 0.27958 respectively and the C.V of the number of employees in grades 1 and 2 are increasing from 1.59563 to 1.61623 and 1.88937 to 1.89126 respectively.

When the rest of the parameters are fixed and the promotion rate parameter (b_1) varies from 9.4 to 10.6, the variance of the number of employees in grade-1 stays constant while it in grade-2 decreases from 0.11353 to 0.01933. The C.V of the number of employees in grade-1 stays constant while in grade-2 it increases from 2.96790 to 7.19187 respectively.

When the remainder of the parameters are fixed and the promotion rate parameter (b_2) changes from 12 to 20, the variance of the number of employees in grade-1 stays constant while in grade-2 it decreases from

Corrosion Management ISSN: 1355-5243
(https://corrosion-management.com/)
Volume 34, Issue 02, July– Dec 2024
SCOPUS

0.01832 to 0.01552, and the C.V of number of employees in grade-1 stays constant while in grade-2 it increases from 7.38825 to 8.02588.

Table 2. Values of $V_1(t)$,$V_2(t)$,$CV_1(t)$ and $CV_2(t)$ for different values of parameters

t	λ_1	λ_2	a_1	a_2	b_1	b_2	Θ	π	$V_1(t)$	$V_2(t)$	$CV_1(t)$	$CV_2(t)$
0.13	2	3	7	5	9	9	0.1	0.9	0.07505	0.01331	3.65017	8.66901
0.14	2	3	7	5	9	9	0.1	0.9	0.09266	0.01124	3.28514	9.43206
0.15	2	3	7	5	9	9	0.1	0.9	0.10921	0.00941	3.02606	10.3037
0.16	2	3	7	5	9	9	0.1	0.9	0.12475	0.00781	2.83122	11.3096
0.16	**3**	3	7	5	9	9	0.1	0.9	0.17384	0.01173	2.39844	9.23427
0.16	**4**	3	7	5	9	9	0.1	0.9	0.22292	0.01564	2.11799	7.99712
0.16	**5**	3	7	5	9	9	0.1	0.9	0.27200	0.01955	1.91740	7.15284
0.16	**6**	3	7	5	9	9	0.1	0.9	0.32109	0.02345	1.76477	6.52962
0.16	6	**4**	7	5	9	9	0.1	0.9	0.32995	0.02345	1.74091	6.52962
0.16	6	**5**	7	5	9	9	0.1	0.9	0.33881	0.02345	1.71799	6.52962
0.16	6	**6**	7	5	9	9	0.1	0.9	0.34767	0.02345	1.69595	6.52962
0.16	6	**7**	7	5	9	9	0.1	0.9	0.35654	0.02345	1.67474	6.52962
0.16	6	7	**7.4**	5	9	9	0.1	0.9	0.37094	0.04024	1.64191	4.98511
0.16	6	7	**7.8**	5	9	9	0.1	0.9	0.38196	0.06753	1.61806	3.84815
0.16	6	7	**8.2**	5	9	9	0.1	0.9	0.39020	0.12117	1.60088	2.87282
0.16	6	7	**8.6**	5	9	9	0.1	0.9	0.39615	0.28033	1.58881	1.88870
0.16	6	7	8.6	**7**	9	9	0.1	0.9	0.39277	0.28013	1.59563	1.88937
0.16	6	7	8.6	**9**	9	9	0.1	0.9	0.38942	0.27994	1.60247	1.89002
0.16	6	7	8.6	**11**	9	9	0.1	0.9	0.38611	0.27975	1.60934	1.89065
0.16	6	7	8.6	**13**	9	9	0.1	0.9	0.38282	0.27958	1.61623	1.89126
0.16	6	7	8.6	13	**9.4**	9	0.1	0.9	0.38282	0.11353	1.61623	2.96790
0.16	6	7	8.6	13	**9.8**	9	0.1	0.9	0.38282	0.05981	1.61623	4.08899
0.16	6	7	8.6	13	**10.2**	9	0.1	0.9	0.38282	0.03403	1.61623	5.42075
0.16	6	7	8.6	13	**10.6**	9	0.1	0.9	0.38282	0.01933	1.61623	7.19187
0.16	6	7	8.6	13	10.6	**12**	0.1	0.9	0.38282	0.01832	1.61623	7.38825
0.16	6	7	8.6	13	10.6	**15**	0.1	0.9	0.38282	0.01735	1.61623	7.59238
0.16	6	7	8.6	13	10.6	**18**	0.1	0.9	0.38282	0.01642	1.61623	7.80474
0.16	6	7	8.6	13	10.6	**21**	0.1	0.9	0.38282	0.01552	1.61623	8.02588
0.16	6	7	8.6	13	10.6	21	**0.2**	**0.8**	0.38282	0.03105	1.61623	5.67516
0.16	6	7	8.6	13	10.6	21	**0.3**	**0.7**	0.38282	0.04657	1.61623	4.63375
0.16	6	7	8.6	13	10.6	21	**0.4**	**0.6**	0.38282	0.0621	1.61623	4.01294
0.16	6	7	8.6	13	10.6	21	**0.5**	**0.5**	0.38282	0.07762	1.61623	3.58928

Corrosion Management
(https://corrosion-management.com/)
Volume 34, Issue 02, July– Dec 2024
ISSN: 1355-5243
SCOPUS

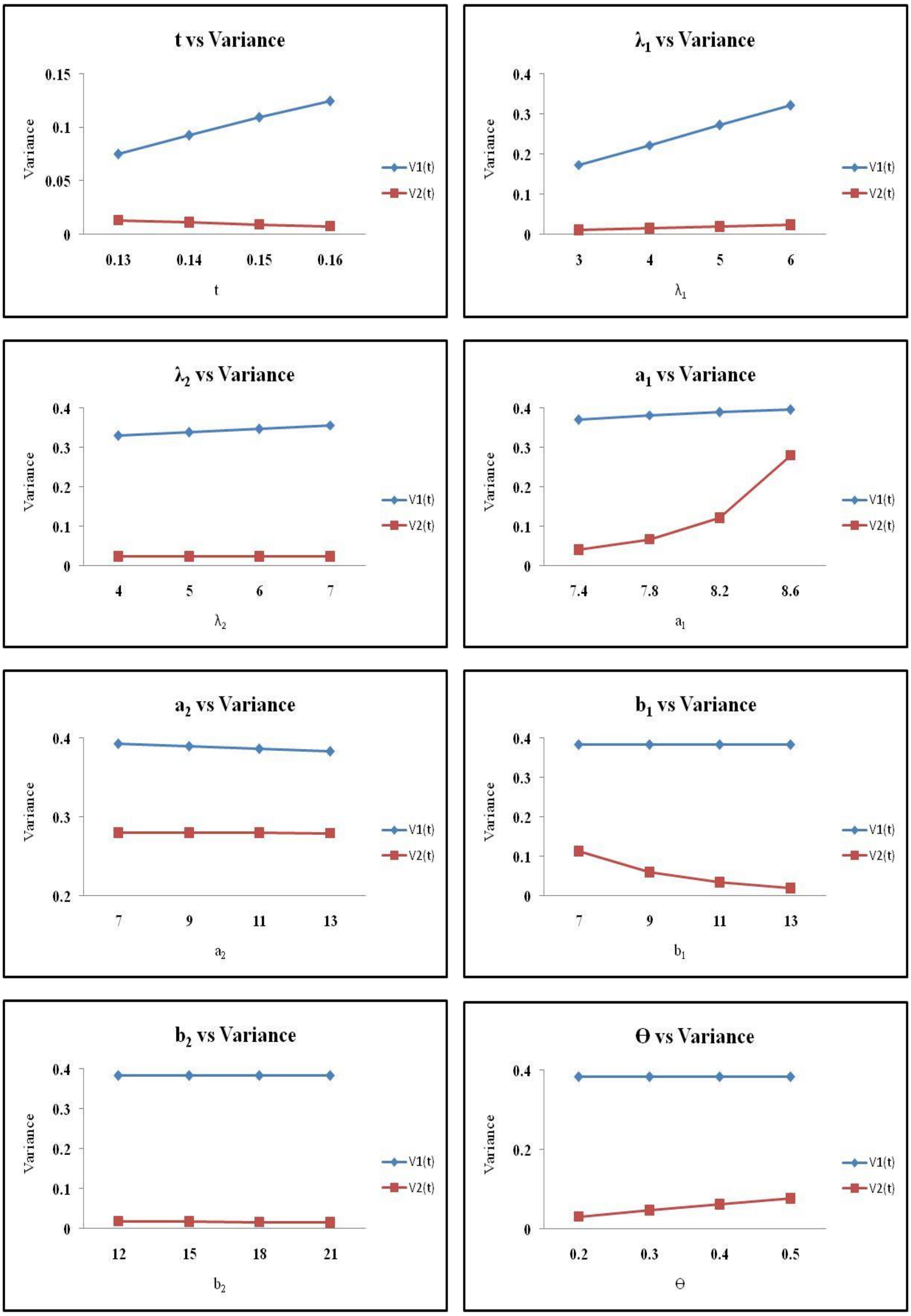

Figure 3a. Relationship between the performance metrics and the parameters

Corrosion Management ISSN: 1355-5243
(https://corrosion-management.com/)
Volume 34, Issue 02, July– Dec 2024
SCOPUS

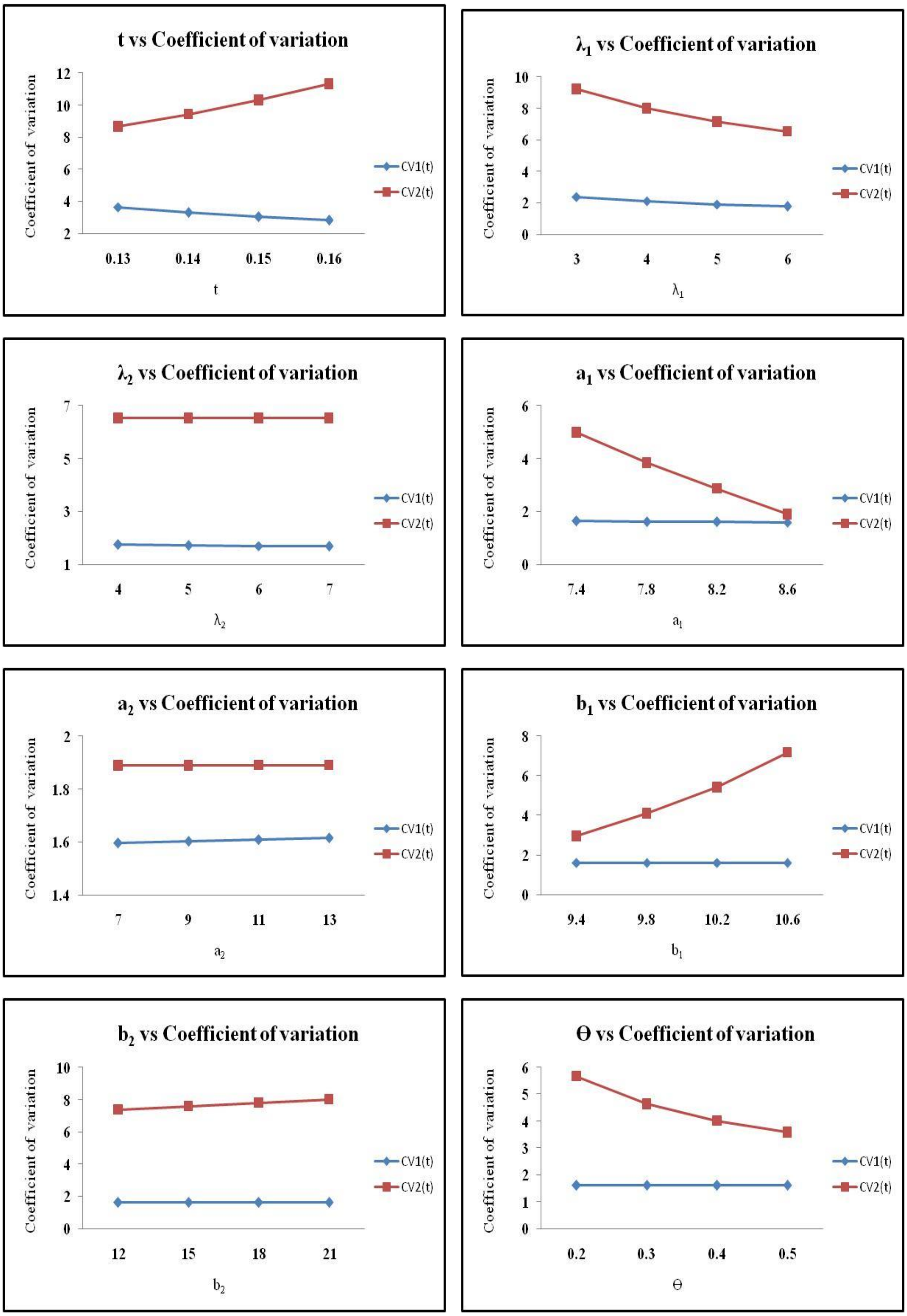

Figure 3b. Relationship between the performance metrics and the parameters

Corrosion Management
(https://corrosion-management.com/)
Volume 34, Issue 02, July– Dec 2024
ISSN: 1355-5243
SCOPUS

When all other parameters are fixed and the promotion rate parameter (θ) varies from 0.2 to 0.5, the variance of the number of employees in grade-1 stays constant while in grade-2 it increases from 0.03164 to 0.07762 respectively. The C.V of the number of employees in grade-1 stays constant while in grade-2 it decreases from 5.67516 to 3.58928.

5. SENSITIVITY ANALYSIS OF THE MODEL

The sensitivity analysis of the model is carried out in relation to the time (t) value, the recruitment rate, the promotion rate and the leaving rate of the grades 1 and 2, as well as all other parameters combined on the average number of employees in the grades 1 and 2, the average waiting time for an employee in the grades 1 and 2 and the variance of the number of employees in grades 1 and 2.

The average number of employees in grades 1 and 2, the average waiting time for an employee in grades 1 and 2 and the variance of the number of employees in grades 1 and 2 are computed and presented in Table 3 for various values of t, λ_1,λ_2 , a_1,a_2, b_1 and b_2 with variation of -15%, -10%, -5% 0%, 5%, 10%, and 15% of the model parameters.

Time (t) has a significant impact on performance measurements. In grade 1, the average number of employees, average employee waiting time and the variance of the number of employees are increasing as t increases from -15% to +15%. In grade 2, the average number of employees, average employee waiting time and the variance are decreasing.

In grades 1 and 2, the average number of employees, the average waiting time for employees and the variance of the number of employees are increasing when the recruitment rate parameter λ_1 increases from -15% to +15%.

The average number of employees, the average employee wait time and the variance of the number of employees are increasing in grade 1 as the recruitment rate parameter λ_2 moves from -15% to +15%, but the performance measures in grade 2 remain same.

The average number of employees, the variance of the number of employees increase and the average waiting time for employees decreases in grade-1 and in grade 2, the average number of employees, average waiting time for employee and employee variance are increasing. When the promotion rate parameter a_1 goes from -15% to +15%.

The average employee count, the average employee wait time and the variance of the number of employees decrease in grade 1 and grade 2 as the promotion rate parameter a_2 moves from -15% to +15%.

When the leaving rate parameter b_1 increases from -15% to +15%, the average number of employees, the average waiting time of employees and the variance of the number of employees in grade-1 remain constant and in grade-2 they are decreasing. When the leaving rate parameter b_2 from -15% to +15% increases, the average number of employees, average waiting time of employee and the variance of the number of employees in grade-1 are unaffected and in grade-2 they are decreasing.

Corrosion Management
(https://corrosion-management.com/)
Volume 34, Issue 02, July– Dec 2024
ISSN: 1355-5243
SCOPUS

Table 3. The values of $L_1(t)$, $L_2(t)$, $W_1(t)$, $W_2(t)$, $V_1(t)$ and $V_2(t)$ for the various values of t, λ_1, λ_2, a_1,a_2,b_1,b_2 and θ.

Parameter	Performance Measures	-15%	-10%	-5%	0%	5%	10%	15%
t=0.2	L1(t)	0.20768	0.22948	0.25000	0.26931	0.28747	0.30456	0.32064
	L2(t)	0.03326	0.02922	0.02558	0.02233	0.01942	0.01683	0.01452
	W1(t)	0.14345	0.14385	0.14416	0.14439	0.14455	0.14464	0.14466
	W2(t)	0.09722	0.09573	0.09428	0.09285	0.09145	0.09007	0.08869
	V1(t)	0.20768	0.22948	0.25000	0.26931	0.28747	0.30456	0.32064
	V2(t)	0.03326	0.02922	0.02558	0.02233	0.01942	0.01683	0.01452
λ1=3	L1(t)	0.23848	0.24876	0.25903	0.26931	0.27958	0.28986	0.30013
	L2(t)	0.01898	0.02010	0.02121	0.02233	0.02345	0.02456	0.02568
	W1(t)	0.14228	0.14298	0.14368	0.14439	0.14510	0.14581	0.14653
	W2(t)	0.09268	0.09274	0.09280	0.09285	0.09291	0.09296	0.09302
	V1(t)	0.23848	0.24876	0.25903	0.26931	0.27958	0.28986	0.30013
	V2(t)	0.01898	0.02010	0.02121	0.02233	0.02345	0.02456	0.02568
λ2=5	L1(t)	0.25974	0.26293	0.26612	0.26931	0.27250	0.27569	0.27888
	L2(t)	0.02233	0.02233	0.02233	0.02233	0.02233	0.02233	0.02233
	W1(t)	0.14373	0.14395	0.14417	0.14439	0.14461	0.14483	0.14505
	W2(t)	0.09285	0.09285	0.09285	0.09285	0.09285	0.09285	0.09285
	V1(t)	0.25974	0.26293	0.26612	0.26931	0.27250	0.27569	0.27888
	V2(t)	0.02233	0.02233	0.02233	0.02233	0.02233	0.02233	0.02233
a1=6.7	L1(t)	0.25225	0.25993	0.26537	0.26931	0.27180	0.27323	0.27373
	L2(t)	0.00172	0.00642	0.01266	0.02233	0.03850	0.07554	0.23037
	W1(t)	0.16421	0.15707	0.15060	0.14439	0.13877	0.13335	0.12844
	W2(t)	0.04895	0.07794	0.08750	0.09285	0.09658	0.10053	0.11010
	V1(t)	0.25225	0.25993	0.26537	0.26931	0.27180	0.27323	0.27373
	V2(t)	0.00172	0.00642	0.01266	0.02233	0.03850	0.07554	0.23037

a2=6	L1(t)	0.27093	0.27039	0.26985	0.26931	0.26877	0.26823	0.2677
	L2(t)	0.02241	0.02238	0.02236	0.02233	0.02230	0.02228	0.02225
	W1(t)	0.14787	0.14669	0.14553	0.14439	0.14327	0.14216	0.14107
	W2(t)	0.09288	0.09287	0.09286	0.09285	0.09284	0.09283	0.09282
	V1(t)	0.27093	0.27039	0.26985	0.26931	0.26877	0.26823	0.26770
	V2(t)	0.02241	0.02238	0.02236	0.02233	0.02230	0.02228	0.02225
b1=7.9	L1(t)	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931
	L2(t)	6.26790	0.11980	0.04679	0.02233	0.01104	0.00456	0.00070
	W1(t)	0.14439	0.14439	0.14439	0.14439	0.14439	0.14439	0.14439
	W2(t)	0.70480	0.11145	0.10074	0.09285	0.08538	0.07522	0.04279
	V1(t)	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931	0.26931
	V2(t)	6.26790	0.11980	0.04679	0.02233	0.01104	0.00456	0.00070
b2=11	L1(t)	0.26770	0.26770	0.26770	0.26770	0.26770	0.26770	0.26770
	L2(t)	0.02316	0.02285	0.02255	0.02225	0.02196	0.02166	0.02137
	W1(t)	0.14107	0.14107	0.14107	0.14107	0.14107	0.14107	0.14107
	W2(t)	0.09928	0.09694	0.09480	0.09282	0.09099	0.08927	0.08766
	V1(t)	0.26770	0.26770	0.26770	0.26770	0.26770	0.26770	0.26770
	V2(t)	0.02316	0.02285	0.02255	0.02225	0.02196	0.02166	0.02137
θ =0.1	L1(t)	0.26770	0.26770	0.26770	0.26770	0.26770	0.26770	0.26770
	L2(t)	0.01891	0.02003	0.02114	0.02225	0.02336	0.02448	0.02559
	W1(t)	0.14107	0.14107	0.14107	0.14107	0.14107	0.14107	0.14107
	W2(t)	0.09266	0.09271	0.09277	0.09282	0.09288	0.09294	0.09299
	V1(t)	0.26770	0.26770	0.26770	0.26770	0.26770	0.26770	0.26770
	V2(t)	0.01891	0.02003	0.02114	0.02225	0.02336	0.02448	0.02559

When the promotion rate parameter θ increases from -15% to +15%, the average number of employees, the average waiting time of employees and the variance of the number of employees in grade-1 are unaffected and in grade-2 they are increasing.

ISSN: 1355-5243

6. COMPARATIVE STUDY OF THE MODELS

This section presents the comparison between the developed model and that of homogeneous Poisson recruitment model. Table 4 shows the performance metrics for both models for various values of t = 0.18, 0.19, 0.20, 0.21, and 0.22.

Table 4 shows that the percentage of variation of the performance measures between the two models also increasing with time. In comparison the model with NHP process is predicting the performance measures more close to the reality in short period of time. It is also evident that every performance metric of the model is significantly impacted by the assumption of NHP recruitment. The performance metrics for the system are significantly impacted by time as well.

Table 4. Comparative study of models with non-homogeneous and homogeneous recruitment

t	Performance Measure	Non-Homogeneous recruitment	Homogeneous recruitment	Difference	Percentage of Variation
t=0.18	$L_1(t)$	0.62123	0.62475	0.00352	0.56343
	$L_2(t)$	0.04528	0.10073	0.05545	55.04815
	$W_1(t)$	0.10034	0.12807	0.02773	21.65222
	$W_2(t)$	0.05714	0.0841	0.02696	32.05707
	$V_1(t)$	0.62123	0.62475	0.00352	0.56343
	$V_2(t)$	0.04528	0.10073	0.05545	55.04815
t=0.19	$L_1(t)$	0.64253	0.65171	0.00918	1.40860
	$L_2(t)$	0.03180	0.08256	0.05076	61.48256
	$W_1(t)$	0.10011	0.12962	0.02951	22.76655
	$W_2(t)$	0.05582	0.08335	0.02753	33.02939
	$V_1(t)$	0.64253	0.65171	0.00918	1.40860
	$V_2(t)$	0.03180	0.08256	0.05076	61.48256
t=0.20	$L_1(t)$	0.66081	0.67598	0.01517	2.24415
	$L_2(t)$	0.02118	0.06716	0.04598	68.46337
	$W_1(t)$	0.09975	0.13103	0.03128	23.87240
	$W_2(t)$	0.05463	0.08272	0.02809	33.95793
	$V_1(t)$	0.66081	0.67598	0.01517	2.24415
	$V_2(t)$	0.02118	0.06716	0.04598	68.46337
t=0.21	$L_1(t)$	0.67638	0.69784	0.02146	3.07520
	$L_2(t)$	0.01291	0.05414	0.04123	76.15441
	$W_1(t)$	0.09928	0.1323	0.03302	24.95843
	$W_2(t)$	0.05354	0.08219	0.02865	34.85826
	$V_1(t)$	0.67638	0.69784	0.02146	3.07520
	$V_2(t)$	0.01291	0.05414	0.04123	76.15441
t=0.22	$L_1(t)$	0.68955	0.71751	0.02796	3.89681
	$L_2(t)$	0.00657	0.04316	0.03659	84.77757
	$W_1(t)$	0.09872	0.13346	0.03474	26.03027
	$W_2(t)$	0.05253	0.08174	0.02921	35.73526
	$V_1(t)$	0.68955	0.71751	0.02796	3.89681
	$V_2(t)$	0.00657	0.04316	0.03659	84.77757

7. CONCLUSION

This paper deals with the stochastic modeling of HR in the organization utilizing the non-stationary Poisson process. The non-stationary Poisson process is capable of characterizing the constituent processes of manpower model such as recruitment, promotion and leaving processes as these processes under non-stationary conditions are time dependent and evolutionary. For example, in big software companies the recruitment in the initial grade is done seasonally and after spending some time in the first grade the employee may be promoted to second grade or leave the organization with certain probability. Utilizing the stochastic calculus the system characteristics of the manpower planning model are derived explicitly. With sensitivity analysis it is noticed that as the input parameters such as recruitment rate, leaving rate and promotion rate have significant influence on the average number of employees in the organization in each grade and average duration of stay of an employee in the organization and other performance measures. The non-homogeneous nature of recruitment, leaving and promotion is clearly visible in the performance measures of the model. This model includes the earlier models with stationarity. Using this model HR manager can take optimal decisions regarding recruitment and can schedule the welfare programs of employees more effectively. This model is also useful for HR analytics in understanding the dynamics of manpower flow in the organization. It is plausible to develop the other non-stationary manpower models where the recruitment is in bulk which will be considered later.

REFERENCES

[1] H. Silcock, "The Phenomenon of the labor turnover," *J.R. Statist. Soc. A*., 117(9), 429 – 440, 1954. https://doi.org/10.2307/2342680.

[2] D. J. Bartholomew, "A multistage renewal process," *J.R. Statist. Soc. B*, 25(1), 150-168, 1963. https://doi.org/10.1111/j.2517-6161.1963.tb00495.x.

[3] D. J. Bartholomew, "The statistical approach to manpower planning," *Statistician*, 20(1),3-2,1971. https://doi.org/10.2307/2987003.

[4] F. I. Ugwuowo and S. I. Mc Clean, "Modeling heterogeneity in a manpower system: a review," *Applied Stochastic Models and Data Analysis*, 16(2), 99-110,2000. https://doi.org/10.1002/1526-4025(200004/06)16:23.0.CO:2-3.

[5] J. Wang, "A review of operations research applications in workforce planning and potential modeling of military training," *DSTO Systems Sciences Laboratory*, 1-37, 2005. http://www.dsto.defence.gov.au/corporate/reports/DSTO-TR-1688.pdf.

[6] Kannan Nilakantan, "Evaluation of staffing policies in Markov manpower systems and their extension to organizations with outsource personnel," *Journal of the Operational Research Society*, 66(8), 1324–1340, 2015. https://www.jstor.org/stable/24505754.

[7] M. Jeeva and N. Geetha, "Recruitment Model in Manpower Planning Under Fuzzy Environment,"*British Journal of Applied Science & Technology*, 3(4), 1380–1390, 2013. https://doi.org/10.9734/BJAST/2014/33825.

[8] Md. Gulzarul Hasan and S. Suhaib Hasan, "Manpower Planning with Annualized Hours Flexibility: A Fuzzy Mathematical Programming Approach," *Decision Making in Manufacturing and Services*, 10(1–2), 5–29,2016. https://doi.org/10.7494/dmms.2016.10.1-2.5.

[9] Md. Gulzarul Hasan, Zoha Qayyum and Syed Suhaib Hasan**, "**Multi-Objective Annualized Hours Manpower Planning Model: A Modified Fuzzy Goal Programming Approach," *Industrial Engineering & Management Systems,* 18(1), 52-66, 2019. DOI:10.7232/iems.2019.18.1.052.

[10] A. A. Osagiede and V. U. Ekhosuehi, "Finding a continuous-time Markov chain via sparse stochastic matrices in manpower systems," *Journal of the Nigerian Mathematical Society*, 34(1), 94-105, 2015. https://doi.org/10.1016/j.jnnms.2014.10.004.

[11] Tames Banyai, Christian Landschützer and Ágota Bányai**,** "Markov-Chain Simulation-Based Analysis of Human Resource Structure: How Staff Deployment and Staffing Affect Sustainable Human Resource Strategy," *Sustainability*, 10(10), 1-21, 2018. https://doi.org/10.3390/su10103692

[12] Bilkisu Maijamaa and Otinya Gabrie, " Decision Making For Recruitment and Promotion Policies Using Linear Programming," *Jurnal Aplikasi Manajemen, Ekonomi dan Bisnis*, 6(2), 48-60,2022. https://doi.org/10.51263/jameb.v6i2.145

[13] A. Vincent Amenaghawon, Virtue U. Ekhosuehi, and Augustine A. Osagiede**, "**Markov manpower planning models: a review," *International Journal of Operational Research*, 46(2), 227-250,2023. https://doi.org/10.1504/IJOR.2023.129157.

[14] C. Anantharaj, **"**Determination of the time to Recruitment in Manpower Planning with Wastages," *Journal of Emerging Technologies and Innovative Research*, 6(6), 337-343,2019. https://www.jetir.org/papers/JETIR1906975.pdf

[15] L. Saral, S. Sendhamizh Selvi and A. Srinivasan, "Estimation of mean time to recruitment for a two graded manpower system with two thresholds, different epoch for exits and correlated inter-decisions under correlated wastage," *International Educational Scientific Research Journal*, 3(3), 1-6, 2017. https://ierj.in/journal/index.php/ierj/article/view/687.

[16] R. Sivasamy, P. Tirupathi Rao and K. Thaga, "Manpower Systems Operating under Heavy and Light Tailed Inter-Exit Time Distributions," *Applied Mathematics*, 5(2), 285-291,2014. https://doi.org/10.4236/am.2014.52029.

[17] G. Ravichandran and K. Srividhya, "Mean square deviation of time to hiring in a Single grade workforce structure with clump of egresses and a random threshold," *Advances and Applications in Mathematical Sciences*, 21(8), 4615-4623,2022. https://www.mililink.com/upload/article/1104987849aams_vol_218_june_2022_a35_p4615-4623_g._ravichandran_and_k._srividhya.pdf.

[18] Ch. Mochammad Munir Rachmana, Menuk Sri Handayani and Sugijanto**,** "The Mediating Role Of Job Satisfaction: The Impact Of Organizational Commitment On Employee Intention To Quit," *Asia Pacific Management and Business Application*, 11(2), 201-220,2022. https://apmba.ub.ac.id/index.php/apmba/article/view/604.

[19] K. Srinivasa rao and Ch. Ganapathi Swamy, "On Two Grade Manpower Model With Duane Recruitment Process," *Journal of Applied Science and Computations*, 6(1), 220-235,2019. https://doi.org/16.10089.JASC.2018.V6I1.453459.1500440.

[20] Ch. Ganapathi Swamy and K. Srinivasa Rao, " A noval application of duane process for modeling two graded manpower systems with direct recruitment in both the grades," *Reliability: Theory & Applications*, 17(2), 38-55, 2022. https://doi.org/10.24412/1932-2321-2022-268-38-55